Private Similarity Search with Sublinear Communication

by

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Abstract

Nearest neighbor search is a fundamental building-block for a wide range of applications. A privacy-preserving protocol for nearest neighbor search involves a set of clients who send queries to a remote database. Each client retrieves the nearest neighbor(s) to its query in the database *without* revealing any information about the query. For database privacy, the client must not learn anything beyond the query answer.

Existing protocols for private nearest neighbor search require heavy cryptographic tools, resulting in poor practical performance or large client overheads. In this thesis, we present the first *lightweight* protocol for private nearest neighbor search. Our protocol is instantiated using two non-colluding servers, each holding a replica of the database. The protocol supports an arbitrary number of clients simultaneously querying the database via these servers. Each query is only a single round of communication for the client and does not require any communication between servers.

If at least one of the servers is non-colluding, we ensure that (1) no information is revealed on the client's query, (2) the total communication between the client and the servers is sublinear in the database size, and (3) each query answer only leaks a small and precisely quantified amount of information about the database to the client, even when the client is acting maliciously.

We implement our protocol and report its performance on real-world data. Our construction requires between 10 and 30 seconds of server processing per query over large databases of 10M feature vectors. Client overhead remained under 10 µs of processing time per query and typically less than 4 MB of communication, depending on parameters.

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"No good fish would go anywhere without a porpoise."

LEWIS CARROLL

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Chapter 1

Introduction

Nearest neighbor search is used in a wide range of online applications, including recommendation engines [36,113], reverse image search [76], image-recognition [79], earthquake detection [120], computational linguistics [86], natural-language processing [101], targeted advertising [99,114], and numerous other areas [73,80,93,99].

In these settings, a server has a database of high-dimensional feature vectors associated with items. Clients send query vectors to the server to obtain the set of items (a.k.a. neighbors) that have similar vectors relative to the issued query. Typically, the client only obtains the items or identifiers (IDs) of the neighbors rather than the feature vectors themselves, as the server may wish to keep the feature vectors private. The IDs of the feature vectors can be documents, songs, or webpages, and therefore all the client requires as output for correct functionality.

For a concrete example, consider a music recommendation engine such as Spotify. The Spotify server holds a database of song feature vectors. Each feature vector can be seen as a concise representation of song attributes—e.g., genre, popularity, user ratings—encoded in a high-dimensional vector space. A Spotify user has a vector of features (the query) representing their musical interests. The goal is to recommend songs the user may find interesting, which should have similar features. This is done using nearest neighbor search to find the ID (e.g., the song) of a vector similar to the query. The client learns the recommended song *without* learning the potentially proprietary feature vector associated with it (beyond what can be implicitly inferred

through similarity with the query).

In the above example, the Spotify database learns exactly which music genres (singers, etc.) the client is interested in. It is not difficult to see that in applications that involve more sensitive user data, similarity search can easily violate user privacy. Such applications include targeted advertising [22,63,100,111,114], biometric data [24,51], medical records [16,106], and DNA data analysis [37,83,91,117]. These applications construct queries from highly personal user information. For example, in online targeted advertising, the browser compiles information on the websites a user has visited into a user profile vector. This profile is the query, and the resulting neighbors are targeted ads with similar features [114]. However, this causes the client to reveal their private interest information to the ad network [22,63,111,114]. In the medical setting, a person's medical history, demographics, and even DNA can be compiled into a query. The resulting neighbors are other people who have similar symptoms, gene sequences, or health conditions [89]. Both regulatory and ethical reasons dictate that such personal information (represented in the query) be kept private from the database.

The potential privacy issues surrounding similarity search have motivated a handful of privacy-preserving protocols [38,67,97,106,121]. However, due to the complexity of the problem, existing protocols are highly inefficient. Prior work either makes use of *heavy* cryptographic tools (e.g., two-party computation and fully-homomorphic encryption) or fails to provide strong privacy guarantees for users (e.g., leak information on the query to the server). See the overview of related work in Section 2.3.

Additionally, existing protocols do not consider *malicious*[1] clients that may attempt to abuse the system to learn more information about the database. The proposed solutions leave open the problem of designing a concretely efficient protocol for private nearest neighbor search, especially when strong security guarantees are required.

The primary contribution of this thesis is the design and implementation of a

[1]Existing work requires secure function evaluation between the client and server, which can be "upgraded" to malicious-security at the cost of computationally expensive transformations based on zero-knowledge proofs [59]. The use of fully-homomorphic encryption is another alternative to providing malicious security but comes at a high efficiency cost.

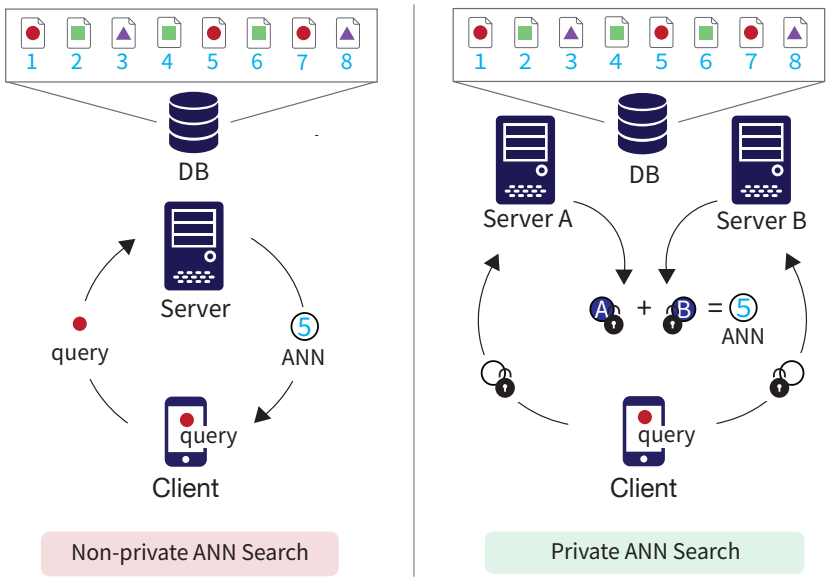

Figure 1-1: Overview of approximate nearest neighbor search and the privacy-preserving protocol considered in this thesis. In the private setting, a client with a query (red dot) interacts with a remote database via two non-colluding servers (Servers A and B). The client combines the responses from both servers to obtain the approximate nearest neighbor ID (in this case the ANN ID = 5) without revealing the query to the servers.

lightweight protocol for private similarity search. Specifically, we provide a private protocol for solving the Approximate Nearest Neighbor (ANN) search problem. Our protocol provides strong security guarantees for both the client and the database. Moreover, our protocol is concretely efficient and requires little communication between the client and the database servers (and no communication between servers). We achieve this without compromising on privacy for the client—nothing is leaked on the client's query. For database privacy, our construction requires some extra database leakage compared to the *ideal leakage* which only reveals the ANN to the client. The additional leakage is minimal and allows us to eschew oblivious comparisons, which are highly inefficient to instantiate [118]. However, we are careful to precisely quantify this extra leakage relative to the ideal functionality. In our analysis, we show that the leakage is at most a constant factor worse than the ideal leakage (asymptotically optimal). Moreover, we ensure that malicious clients cannot abuse the protocol to learn more than an honest client would have from a query.

1.1 Setting

We operate in the following model (see Figure 1-1 for a simplified illustration). Fix a database $\mathcal{DB}$ containing a set of n feature vectors $\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}$ and corresponding IDs $\mathsf{ID}_1, \ldots, \mathsf{ID}_n$. A client has a query vector $\boldsymbol{q}$. The client must learn only the ID(s) of the nearest neighbor(s) relative to $\boldsymbol{q}$ under some similarity (or distance) metric. This is the standard setup considered in prior work [38,67,97,106,121].

For client privacy, the protocol must not leak any information about $\boldsymbol{q}$ to the database servers. For database privacy, the protocol must leak as little as possible on $\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}$ to the client. Observe that perfect database privacy (i.e., no database leakage) is unattainable because the client *must* learn at least one ID corresponding to a neighboring vector in the database, which indicates that the vector associated with the ID is similar to the query. We therefore focus on minimizing extra leakage of the database to the client. That is, leaking as little as possible beyond what can be implicitly inferred from the ID of the nearest neighbor.

Challenges. Due to the nature of the ANN search problem, privacy-focused solutions use some form of (oblivious) distance comparisons between the query and the feature vectors in the database. Oblivious comparisons [118] require the use of heavy cryptographic tools (e.g., two-party computation or fully-homomorphic encryption). Two-party computation introduces heavy communication costs, typically 1-6 GB per ANN query [38]. Solutions based on fully-homomorphic encryption introduce high computational overhead for the server (hours or even days of processing time [106,121]), resulting in significant response latency and making them entirely impractical on larger datasets. In this thesis, we begin by asking the following question:

> *Is it possible to design a private similarity search protocol that is concretely efficient for both the client and the servers while preserving query privacy* ***and*** *database privacy?*

We answer this question in the affirmative. We overview our approach in Section 1.2.

1.2 Approach

We begin by making several observations about the ANN search problem from both an algorithmic and privacy-preserving perspective.

We redesign the standard locality-sensitive-hashing based data structure for ANN search with the goal of avoiding oblivious comparisons, which we identify as the efficiency bottleneck of prior work. We achieve this by replacing brute-force comparisons with a radix-sort [74] inspired approach for extracting the nearest neighbor. We then show how to find the nearest neighbor(s) using this new data structure through a novel privacy-preserving protocol. To do so, we use distributed point functions [56] to privately query our data structure and then apply an efficient leakage-suppressing transformation (introduced in Section 4.1.2) that prevents clients from learning more than one ID per query. Finally, for efficiency, we apply partial batch retrieval (a new spin on batch-PIR [15], which we introduce in Section 4.3.2),

We show that our protocol is (1) *accurate* through an extensive theoretical analysis and empirical evaluation, (2) *private* by analyzing the security properties of our protocol with respect to client and database privacy, and (3) *efficient* in terms of concrete server-side computation and client-server communication.

1.3 Contributions

In summary, this thesis makes the following four contributions:

1. design of a single-round protocol for privacy-preserving ANN search, achieving sublinear communication and concrete efficiency,
2. leakage analysis with quantifiable database privacy, which we show asymptotically matches the ideal functionality,
3. security against malicious clients that may deviate from protocol in an attempt to abuse leakage and,
4. an open-source implementation [2] which we evaluate on real-world data with millions of feature vectors.

Chapter 2

Background

We begin by describing the standard (non-private) approach to approximate nearest neighbor search based on locality-sensitive hashing. Even outside of a privacy-preserving context, nearest neighbor search in higher dimensions ($d \geq 10$) requires tolerating approximate results to achieve efficient solutions [85]. In later chapters, we will transform the ideas from non-private ANN search into a private protocol instantiated between a client and two servers holding replicas of the database.

2.1 Locality-sensitive Hashing

The approximate nearest neighbor search problem is solved (efficiently) using hashing-based techniques that probabilistically group similar feature vectors together (see survey of Andoni et al. [13]). Approximate solutions based on Locality-sensitive Hashing (LSH) provide tunable accuracy guarantees and only require examining a small fraction of feature vectors in the database to find the approximate nearest neighbor(s).

LSH families are defined over a distance (or similarity) metric such as Euclidean distance [8,46] or Jaccard similarity [32]. LSHs have the property that feature vectors close to each other in the metric space hash to the same value with good probability. Formally, for a fixed feature vector space $\mathcal{D}$, output space $\mathcal{R}$, and a distance metric Δ, an LSH family is defined as follows.

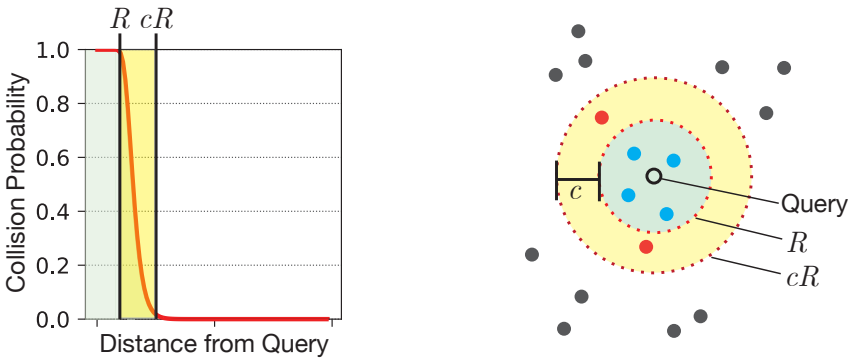

Figure 2-1: Visualization of the nearest neighbor search problem based on LSH. **Left:** collision probability of an LSH-based data structure as a function of the distance between the query and points in the database. **Right:** points colliding with the query in the ANN data structure. Points within distance R have a high probability of colliding with the query whereas points within distance R and cR from the query have a lower probability of colliding. The approximation factor $c > 1$ determines the quality of the results; typically $c = 2$ in practical applications.

Definition 1. A family of hash functions $\mathcal{H} := \{\mathsf{h} : \mathcal{D} \to \mathcal{R}\}$ is (R, cR, p_1, p_2)-sensitive for distance metric Δ if for any pair of vectors $\boldsymbol{v}, \boldsymbol{q} \in \mathcal{D}$,

$$\textbf{if } \Delta(\boldsymbol{v}, \boldsymbol{q}) \leq R \textbf{ then } \Pr[\mathsf{h}(\boldsymbol{v}) = \mathsf{h}(\boldsymbol{q})] \geq p_1,$$

$$\textbf{if } \Delta(\boldsymbol{v}, \boldsymbol{q}) \geq cR \textbf{ then } \Pr[\mathsf{h}(\boldsymbol{v}) = \mathsf{h}(\boldsymbol{q})] \leq p_2,$$

where $R < cR$ and $p_1 > p_2$.

Remark 1. Note that a LSH family is usually combined with a universal hash function to map to a fixed output size [46]. Without loss of generality, we assume that the LSH is also a universal hash with negligible collision probability.

2.2 Approximate Nearest Neighbor Search

In this work we adapt the data structure of Gionis et al. [57], which is the standard way of solving the ANN search problem using LSH (see survey of Andoni et al. [13]). The data structure consists of two algorithms: Build and Query, described in Figure 2-2.

Parameters & Guarantees. The probability that a nearest neighbor collided with the query in a subset of hash tables can be made arbitrarily high (by tuning

LSH-based ANN search data structure

BUILD $(\mathcal{DB}, \mathcal{H}, L) \to (\mathcal{T}_1, \ldots, \mathcal{T}_L)$.

Input:

- set of n vectors $\mathcal{DB} = \{\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}\}$,
- and LSH family $\mathcal{H}$ (Definition 1),
- number of hash tables L.

Output:

- hash tables $\mathcal{T}_1, \ldots, \mathcal{T}_L$.

Procedure:

1: Sample L random functions $\mathsf{h}_1, \ldots, \mathsf{h}_L$ from $\mathcal{H}$.
2: Use hash function h_i to construct hash table $\mathcal{T}_i$ by hashing each vector $\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}$.
3: Output set of L hash tables $\mathcal{T}_1, \ldots, \mathcal{T}_L$.

QUERY $(\mathcal{T}_1, \ldots, \mathcal{T}_L, \mathsf{h}_1, \ldots, \mathsf{h}_L, \boldsymbol{q}) \to \mathsf{ID}$.

Input:

- query vector $\boldsymbol{q}$,
- L hash tables,
- and functions $\mathsf{h}_1, \ldots, \mathsf{h}_L$.

Output:

- approximate nearest neighbor to $\boldsymbol{q}$ in $\mathcal{DB}$.

Procedure:

1: Compute bucket key $l_i \leftarrow \mathsf{h}_i(\boldsymbol{q})$ for $i \in \{1, \ldots, L\}$.
2: Retrieve bucket $\mathcal{B}_{l_i}$ in $\mathcal{T}_i$ under key l_i (if non-empty).[a]
3: Set $\mathcal{C} := \mathcal{B}_1 \cup \cdots \cup \mathcal{B}_L$.
4: Find j such that $\boldsymbol{v_j} \in \mathcal{C}$ and $\Delta(\boldsymbol{v_j}, cR) \leq cR$, via brute-force comparisons.
5: **if** no such j exists, output 0; **else** output j.

[a] Note that by the properties of LSH, the query will collide with probability proportional to the relative distance from other vectors.

Figure 2-2: LSH-based ANN search data structure of Gionis et al. [57].

parameters). As such, BUILD and QUERY ensure that the nearest neighbor is found with high probability (see Figure 2-1). In practice, one must query $L \approx \sqrt{n}$ tables to find a near neighbor in expectation [13].

Remark 2. Following prior work [9,13,57], we described a setting for finding *one* approximate nearest neighbor to a given query. However, the data structure and definitions naturally generalize to the approximate k-nearest neighbor problem, where k neighbors are retrieved per query [57, Figure 2].

2.3 Related Work

Existing works on privacy-preserving similarity search either use heavy cryptographic tools (e.g., general secure function evaluation instantiated using two-party computation and fully-homomorphic encryption) or provide poor privacy guarantees for either the client or the database.

	Comm.	Comp.	Rounds	Malicious sec.	Tools	Efficiency
SFR20 [106]	$\log n$	n	1	✓	FHE	●○○○[1]
SANNS [38]	n/k	n	n	✗	2PC	●●○○
IW06 [67]	$\sqrt{n}$	n	n	✗	2PC	N/A
QA08 [97]	n	n	n	✗	2PC	●●○○
ZS21 [121]	$\log n$	n^2	1	✓	FHE	●●○○[3]
Chapter 4	$\sqrt{n}$	n	1	✓	DPF	●●●●

Table 2.1: Comparison to related-work. n is the database size. We assume dimension $d \in \mathsf{polylog}(n)$ [57]. Constant and $\log n$ factors suppressed for readability.

[1]Uses fully-homomorphic encryption; takes several hours of computation to evaluate with $n \approx 1000$.
[2]SANNS uses k-means clustering to reduce communication by a constant factor k.
[3]Uses fully-homomorphic encryption; takes over 20 minutes to evaluate with $n \approx 250$.

To the best of our knowledge, all existing works on privacy-preserving similarity search (with the exception of [106,121] which use fully-homomorphic encryption) only consider honest-but-curious clients and servers and require many rounds of communication. While generic techniques for upgrading to active security in two-party computation exist [81], they are computationally expensive and often considered impractical. Our protocol is the first to assume fully malicious clients, which solves the challenge left open by prior approaches [38]. A comparison of the following approaches is summarized in Table 2.1.

Two-party computation based approaches. Indyk and Woodruff [67] investigate nearest neighbor search between two parties under the Euclidean distance metric. They show a $\tilde{O}(\sqrt{n})$ communication protocol for finding an approximate *near* (as opposed to *nearest*) neighbor to a query. Their techniques rely on black-box two-party

computation. This makes them only asymptotically efficient (they do not provide an implementation or any concrete efficiency estimate). However, their protocol shares some to similarity ours. Specifically, they tolerate some precisely quantified leakage, which they argue can be a suitable compromise in favor of efficiency gains.

More recently, Chen et al. [38] design and evaluate SANNS, a system for approximate nearest neighbor search that uses oblivious RAM, garbled circuits, and homomorphic encryption. Their solution combines heuristic k-means clustering techniques to reduce overhead of two-party computation of computing oblivious comparisons by a constant factor (i.e., $\frac{1}{k}$). However, they still require asymptotically linear communication, since k is typically small. They leave open the possibility of using locality-sensitive hashing to provide *provable* guarantees, as we do in this thesis.

By assuming data-dependent clusters, SANNS is able to forgo a linear scan over the database when evaluating oblivious distance comparisons. While their implementation is efficient over high-bandwidth connections (40MB/s to 7GB/s bandwidth), such throughput is often too high to be practical on real-world connections (especially on mobile networks, which can be over 30× slower [108]). Moreover, SANNS requires the data to be clusterable to reduce communication. Our protocol, in contrast, does not make any assumptions on the input data to achieve sublinear communication. We show in our evaluation (Chapter 6) that our protocol requires between 484–1511× less communication compared to SANNS, and microseconds of processing on the client.

Qi and Atallah [97] present a protocol for privacy-preserving nearest neighbor search in the honest-but-curious setting with two parties. In contrast to us, they assume each party (i.e., server) has a database that is private from the other party. Queries are computed over the union of both databases. Their protocol uses secure two-party computation to compute oblivious comparisons and requires linear communication in the database size. Qi and Atallah [97] do not provide an implementation or any concrete efficiency estimates for their protocol.

Fully-homomorphic encryption based approaches. Shaul et al. [106] present a protocol based on fully-homomorphic encryption, requiring several hours of computation time to answer queries over small (1000 item) databases. While this results in

both a single-round protocol and tolerates malicious clients, it is not practical for large databases. Their implementation requires between three and eight hours (parallelized across 16 cores) to compute the nearest neighbors on small datasets ranging between 1,000 and 4,000 feature vectors.

Zuber and Sirdey [121] implement a secure nearest neighbor classifier using (threshold) fully-homomorphic encryption with applications to collaborative learning and nearest neighbor search. Their approach requires over one hour of server processing time to compute a query answer over a small database of approximately 500 feature vectors. As such, their protocol is not scalable beyond databases containing a few thousand feature vectors.

Partially-private approaches. Not directly related to privacy, Aumüller et al. [19] introduce distance-sensitive hashing which they show can be beneficial to reducing information leakage between hashes. However, their security guarantees are not formally defined and their approach provides a trade-off between privacy and accuracy, leaking information about the client's query and the database simultaneously.

Riazi et al. [103] likewise explore LSH as a means of trading-off privacy with accuracy, with more accurate results revealing more information on both the query and the database. They make use of two-party computation to instantiate a garbled circuit for the purpose of securely evaluating locality-sensitive hashes without revealing the description of the hash function. While this reduces some leakage, their approach still reveals information on the query and the database.

In a similar vein, Boufounos and Rane [29] develop a binary embedding (locality-sensitive hash) that preserves privacy when finding similar feature vectors in a remote database. Their technique is less general compared to [19,103]. Boufounos and Rane [29] do not provide a formal security analysis of their nearest neighbor search protocol based on their secure embedding. Analyzing their protocol, we found that (1) some partial information on the query is inadvertently leaked to the server, and (2) the client learns the distances from its query to all near-neighbors, resulting in significant database leakage. No implementation or any concrete runtime estimates were provided for their protocol.

Servan-Schreiber et al. [104] develop AdVeil, a system for privacy-preserving targeted advertising. In AdVeil, ad targeting is performed using single-server private information retrieval and locality-sensitive hashing to find relevant ads (nearest neighbors) for a user from a remote ad database. While user privacy is guaranteed in their targeting scheme, database privacy is not.

Dauterman et al. [47] develop a two-server encrypted search system called Dory for reverse-index queries to a database (i.e., exact keyword matching). Their goal is solving metadata-private *exact* keyword search where only privacy for the client is considered. We believe that our techniques can be directly applied to extend Dory to support fuzzy-keyword reverse-index search.

Client-outsourced data. Other systems for privacy-preserving nearest neighbor search (e.g., [16,49,64,77,116]) focus on a setting where a client *outsources* the data to a remote database in a way that it can later privately query for nearest neighbors. As such, they do not consider malicious clients nor are they interested in preserving the privacy of item attributes in the database. We consider this setting as orthogonal.

Chapter 3

Private Approximate Nearest Neighbor Search

We adapt the data structure of Gionis et al. [57] (see Chapter 2) into a privacy-preserving protocol between a client with query vector $\boldsymbol{q}$ and two servers with access to replicas of the database. We begin by describing the ideal functionality of privacy-preserving approximate nearest neighbor search.

3.1 Notation

We denote by $\mathcal{DB}$ the database of vectors and their IDs. We let n be the total number of d-dimensional vectors in $\mathcal{DB}$. A vector is denoted in bold as $\boldsymbol{v}$ where the ith coordinate of $\boldsymbol{v}$ is denoted by $\boldsymbol{v}_i$. We denote the ith value of a set S by $S(i)$. A distance metric (e.g., Euclidean distance) is denoted Δ, where threshold distances R and cR are as defined in Section 2.2. We let $\mathbb{F}$ denote any prime-order finite field (e.g., integers mod a prime p). A secret-share of a value $v \in \mathbb{F}$ is denoted using bracket notation as $[v]$. Coordinate-wise secret-shares of a vector $\boldsymbol{v}$ is denoted $[\boldsymbol{v}]$. Variable assignment are denoted by $x \leftarrow y$, where $x \xleftarrow{R} S$ denotes a random sample from the set.

3.2 Ideal Functionality

The ideal ANN search functionality is described in Functionality 1. The functionality takes as input the public parameters and query $\boldsymbol{q}$ to output the ID of the nearest neighbor relative to $\boldsymbol{q}$ to the client. The servers obtain no output.

Without loss of generality, we assume the ID of the ANN is the *index* of the ANN for some canonical ordering of the feature vectors in the database. We let ID = 0 when no nearest neighbor exists.

Functionality 1: Private ANN Search

Public Parameters:

- distance metric Δ,
- database size n,
- feature vector dimensionality d,
- maximum nearest neighbor radius R_{max}.

Inputs:

- Client: query $\boldsymbol{q} \in \mathbb{R}^d$,
- Server(s): database $\mathcal{DB} := \{\boldsymbol{v_1}, \ldots, \boldsymbol{v_n} \mid \boldsymbol{v_i} \in \mathbb{R}^d\}$.

Procedure:

1: $\boldsymbol{v_j} \leftarrow$ nearest neighbor to $\boldsymbol{q}$ in $\mathcal{DB}$ via brute-force search.
2: **if** $\Delta(\boldsymbol{v_j}, \boldsymbol{q}) > R_{\mathsf{max}}$ **then**
output 0 to the client and $\perp$ to the servers.
3: **else** output j to the client and $\perp$ to the servers.

Restricting the problem. Note that an LSH-based algorithm will only return an answer that is within distance cR of the query. We formalize this by assuming that the ideal functionality outputs the nearest neighbor that is also a *near*[1] (distance less than cR away from the query) neighbor. While it is possible to imagine contrived databases where the nearest neighbor is *not* also a near neighbor, most practical instances of the problem impose this additional restriction (returning no neighbor if the nearest neighbor is beyond a threshold distance from the query) because points beyond some threshold are effectively unrelated to the query. To this end, the ideal functionality is defined to reveal the ID of the nearest neighbor (if one exists) within a fixed distance $cR = R_{\mathsf{max}}$ from the query. Following Beyer et al. [26], we define two

[1]This name comes from the *near* neighbor search problem [65][6].

quantities D_{max} and D_{min} to be the maximum and minimum distance between any two points, respectively. Because the distance between any two vectors is at most D_{max}, it suffices to have $R_{\mathsf{max}} < D_{\mathsf{max}}$ to formalize the above restriction.

3.3 Threat Model and Guarantees

Our protocol is instantiated with two non-colluding servers and an arbitrary number of clients. Clients query the servers to obtain an ID of the ANN. We do not require any communication between servers when answering queries.

3.3.1 Threat Model

- No client is trusted by either server. Clients may deviate from protocol, collude with other clients, or otherwise behave maliciously to learn more about the database.
- No server is trusted by any client. One or both servers may deviate from protocol in an attempt to obtain information on a client's query or the resulting nearest neighbor.

3.3.2 Assumptions

Our core assumption, required for client privacy, is that the two servers do not collude with each other. Additionally, we require black-box public-key infrastructure (e.g., TLS [102]) to encrypt communication over the internet. Implicitly, we assume that one-way functions exists when instantiating distributed point functions [30].

3.3.3 Guarantees

Under the above threat model and assumptions, the protocol provides the following guarantees.

Correctness. If the client and servers both follow protocol, then the client obtains the ID of the ANN with respect to its query. The result is guaranteed to have the

same approximation accuracy of standard, non-private data structures for ANN search, and has tunable accuracy guarantees.

Client privacy. If the servers do not collude, then neither server learns any information on the client's query, even if one or both servers arbitrarily deviate from protocol.

Bounded leakage. Each query answer is guaranteed to leak a small (and tightly bounded) amount of information over the ideal functionality, even if the query is maliciously generated by the client. We provide a precise definition and in-depth analysis of this leakage in Chapter 5.

3.4 Main Ideas

To introduce privacy, as required for the client *and* the database, we make several changes to the data structure of Figure 2-2 (BUILD and QUERY). A simple strawman protocol with *client* privacy (but no database privacy) can be realized by applying well-known techniques in Private-information Retrieval (PIR) to privately obtain the answer to QUERY (see [40,41,54]). Specifically, PIR allows a client to privately retrieve a specified object from a database without revealing which object was retrieved, which naturally generalizes to retrieving buckets from a hash table [41]. While PIR solves the client privacy problem, it provides no database privacy. The client learns $\sqrt{n}$ (see Section 2.2) feature vectors from the database *per query*, which results in unacceptable leakage.

3.4.1 The Challenge with Database Privacy

The primary challenge in reducing database leakage comes from preventing the client from learning extra vectors in the candidate set. This is non-trivial to do given that the standard approaches to removing false-positive candidates (points farther than cR from the query) require some form of direct distance comparisons. In the private setting, these become *oblivious* comparisons (a comparison between secret-

shared values), which in turn require *heavy* cryptographic techniques (e.g., garbled circuits [118]). The state-of-the-art approach for privacy-preserving ANN search (SANNS [38]) prunes candidates by using an expensive two-party computation, which requires several gigabytes of communication between the client and database server.

The insight that we exploit to overcome this challenge is that LSH can be used directly to accomplish the same goal of pruning false-positives. We then apply a trick inspired by radix sorting [74] to extract the *nearest* neighbor, fully removing the need for direct comparisons between vectors. Our new data structure is slightly less efficient when viewed from an *algorithmic* perspective (i.e., when compared to the data structure of Figure 2-2). However, this is not a problem for us given that oblivious comparisons are the primary bottleneck in a *privacy-preserving* setting. We elaborate on this observation in the next section.

3.4.2 Reframing the Problem

LSH-based ANN search is typically optimized to minimize the number of hash tables (L) and the size of the candidate set for each query. However, removing false-positives via brute-force comparisons is relatively "cheap" from a computational standpoint while hash table lookups are relatively expensive. Therefore, LSH-based ANN search is typically tuned to retrieve as many (reasonable) candidates as possible from each table. The extra candidates are then pruned via brute-force comparisons.

The privacy-preserving setting requires different priorities. First, note that it is not possible to perform only one lookup per hash table. To preserve privacy, *all* hash table buckets must be "touched" by the database server(s) to avoid revealing information on the client's query. This is the lower bound on private information retrieval [58]. Otherwise, the servers learn that the client's hash does *not* correspond to any untouched bucket. This is exactly why existing solutions for privacy-preserving ANN require $O(n)$ communication between the client and server, or alternatively, the use of fully-homomorphic encryption (which requires linear server work). As such, we cannot hope to have sublinear (in n) work for the servers when answering client queries.

With this observation in mind, we note that the optimal algorithmic LSH parameters might not in fact be optimal for privacy-preserving data structures. We therefore re-design and re-tune the ANN search data structure of Figure 2-2 for the purpose of avoiding comparisons between vectors.

3.4.3 Eliminating Oblivious Comparisons

We note that false-positives (values farther than cR from the query) are always pruned. One idea to remove comparisons is to prevent values that will be pruned from being added to the candidate set in the first place. Then, *any* candidate element that collides with the query is guaranteed to be at most cR away from the query. The challenge, however, is that all known LSH-based data structures require *filtering* all false-positives from the candidate set. Instantiating this filtering phase privately would require performing oblivious comparisons which would immediately make the protocol *not* lightweight. We observe that we can exploit the inherent properties of LSH to design a protocol that avoids oblivious comparisons entirely by (nearly) eliminating all false-positives. More precisely, by tuning the parameters of the ANN search data structure, and the LSH functions it uses, we can bound the probability of false-positives in the candidate set to any $0 < \delta < 1$.

To the best of our knowledge, such techniques have not been explored yet from an *algorithmic* angle given that eliminating false-positives via brute-force comparison is computationally cheaper *outside* of privacy-preserving contexts; as explained in Section 3.4.2.

3.4.4 Bounding False-positives

A standard result in LSH theory is that a (R, cR, p_1, p_2)-sensitive hash family can be "amplified" to result in a (R, cR, p'_1, p'_2)-sensitive hash family, where $p'_1 = p_1^k$ and $p'_2 = p_2^k$. This is done by simply concatenating the outputs of k independent (R, cR, p_1, p_2)-sensitive hash functions. Amplification is generally used to reduce the size of the candidate set [13], however, we take this approach to its extreme. For a larger value

of k, fewer collisions will occur. On the other hand, when considering collisions that do occur, they are more likely to be true positives. As a result, with a sufficiently large k, we can significantly reduce false positives and compensate for the smaller p_1' by increasing the number of tables L. We capture this idea in Proposition 1.

Proposition 1. Fix failure probability $\delta > 0$, and any (p_1, p_2, R, cR)-sensitive hash family where $\rho = \frac{\log(p_1)}{\log(p_2)} < \frac{1}{2}$. There exists a data structure solving the cR-approximate near neighbor problem in $O(n^{\rho'})$ time and $O(n^{1+\rho'})$ space, where $\rho < \rho' < 1$. This data structure returns the true cR-approximate nearest neighbor with probability $1 - \delta$, without requiring brute-force distance comparisons between vectors.

Proof. The LSH data structure of Section 2.2 (Figure 2-2) achieves asymptotic space and query time $O(n^{1+\rho})$ and $O(n^{\rho})$, respectively [13]. Consider the data structure described in Figure 2-2 where QUERY is modified to a *random* element $\in \mathcal{C}$ (if $\mathcal{C} \neq \varnothing$) instead of an element that is guaranteed to be within distance cR. We need to bound the success probability of this random element being within cR of the query to $1 - \delta$. By Definition 1 (LSH), we have that $p_1 > p_2$. Therefore, there exists a "gap" between the probability of a true positive and a false positive collision, which can be amplified exponentially in k. First, observe that the probability of selecting a false-positive at random from $\mathcal{C}$, when the database size is n, is bounded by

$$\frac{n \cdot p_2^k}{p_1^k + n \cdot p_2^k} < n \cdot \frac{p_2^k}{p_1^k}, \tag{3.1}$$

which we further need to bound by δ. Since we have that $p_1 > p_2$ (Definition 1), it follows that $p_2/p_1 < 1$. Therefore, when

$$k \geq \left\lceil \frac{\log(n) + \log(1/\delta)}{\log(1/p_2) + \log(p_1)} \right\rceil$$

we get that (3.1) is bounded by δ, which leads to success probability $1 - \delta$ of selecting a true-positive at random (when it is contained in the candidate set). We contrast this to "vanilla" LSH-based data structures (e.g., Figure 2-2) where $k \geq \left\lceil \frac{\log(n)}{\log(1/p_2)} \right\rceil$ [13]. We note that the values are asymptotically similar. The new data structure results in

the elimination of brute-force comparison while still preserving accuracy guarantees. Finally, to prove space and query time, we note that

$$L = \left\lceil p_1^{-k} \right\rceil = n^{\frac{\rho}{1-\rho}} \cdot \delta^{\frac{1-\rho}{\rho}}.$$

Therefore, we get $\rho' < 1$ when $\rho < \frac{1}{2}$. Following the LSH-based ANN search data structure of Chapter 2, the asymptotic query time is $O(L)$ and the space is $O(n \cdot L)$. ■

When is $\rho < \frac{1}{2}$? The restriction $\rho < 1/2$ (required for sublinearity in Proposition 1) is met for many common distance metrics when the approximation factor $c \geq 2$. We cite various results for when these conditions are met. We note that these values are for *worst-case* theoretical guarantees; in practice, ρ is much smaller (see Chapter 6). For Euclidean distance, Andoni and Indyk [8] show $\rho < \frac{1}{c^2} = \frac{1}{4}$ with $c = 2$. For angular distance, it is possible to apply the Andoni and Indyk [8] by normalizing, as explained in in [11]. For any p-stable distribution (including ℓ_1 norm) Datar et al. [46] show $\rho < \frac{1}{c} = \frac{1}{2}$ when $c = 2$. For the ℓ_1-norm specifically, Motwani et al. [88] show $\rho < \frac{1}{2c} = \frac{1}{4}$ when $c = 2$. Hamming distance can be embedded into Euclidean space (see Aumüller et al. [18]). Alternatively, [119, Corollary 3.10] show $\rho < \frac{1}{c} = \frac{1}{2}$ for hamming distance directly.

Bounded false-positives (in the worst case). The consequence of Proposition 1 is that while we can bound false-positives to any δ, this comes with the cost of increasing the number of tables L, since $L = \lceil p_1^{-k} \rceil$. Because k is a function of the LSH sensitivity, we need to ensure that the difference between p_1 and p_2 is sufficiently large to result in reasonable values of k and L. We describe such LSH families in Section 3.4.5. In our evaluation (Chapter 6), we show that on real data, we can have false positive probability less than 0.05 with $k = 2$. With Proposition 1 we can guarantee that all collisions are within cR from the query, with high probability.

3.4.5 Efficient LSH for Euclidean Distance

In this section we describe how to instantiate Proposition 1 efficiently for Euclidean distance ANN search. We first note that Proposition 1 is an upper bound and assumes worst-case data (all points are between R and cR from the query). In practice, there are LSH functions that can achieve very small p_2 without needing a large k. We take another look at Proposition 1. For a large enough $k > d$, the region of space defined by the set $\{x \mid \mathsf{h}(x) = \mathsf{h}(y)\}$ for a fixed y becomes bounded. Specifically, there exists a bounding distance b such that

$$\Pr[h(x) = h(y) \wedge \|x - y\| > b] < \delta.$$

For a false-positive rate of δ, it suffices to scale the space so that the bounding distance b is equal to cR. This scaling decreases p_1 as explained in the proof of Proposition 1. For better efficiency, we observe that it is possible to use LSH functions that *inherently* have bounded regions, such as the Leech lattice based LSH of Andoni and Indyk [8]. By using structure rather than randomness they can achieve a larger p_1 for the same p_2 as one might expect from $k \approx d$. We briefly describe a lattice-based LSH family for Euclidean distance next.

Lattice-based LSH. A simple lattice-based locality sensitive hash family for Euclidean distance is as follows. Choose an infinite set of points in $\mathbb{R}^d$, and let the hash of x be the closest point of this set. For example, if we consider the set of points with integer coordinates, we can efficiently find the hash by rounding the coordinates of x. Lattice based hashes allow us to bound the error distance on p_2; with the integer points, it is clear that $b \leq \sqrt{d}$. Furthermore, we can reduce b to b' by returning $\perp$ if the distance $\|x - h(x)\|$ is not less than $b'/2$. However, the set of integer points is not optimal in that the error increases greatly for the higher dimensional data we wish to use it on. For an efficient implementation, we use the Leech lattice (as in [8, Appendix B]), which has $b \leq \sqrt{2}$ for $d = 24$. See Conway et al. [43] or Conway and Sloane [42] for more information on the lattice structure.

3.4.6 Finding the Nearest Neighbor

We are now left with the problem of finding the *nearest* neighbor within the set of all cR-neighbors. Our idea for doing so is based on a bucketing technique of Har-Peled et al. [65], which resembles radix sorting [74]. A radix sort does *not* perform direct comparisons, which aligns with our goals. We recursively apply the data structure of Proposition 1 on a series of increasing neighbor radii, retrieving a set of candidates from each radius [65]. The client then selects a result from the first non-empty candidate set. We illustrate this process in Figure 3-1.

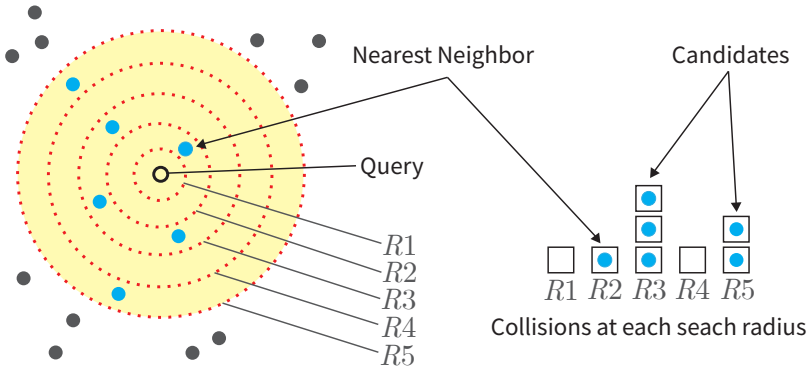

Figure 3-1: Illustration of using multiple radii (R) to search for the nearest neighbor.
Left: Each of the dotted regions represents a different hash function radius.
Right: The candidate with the smallest R_i is the nearest neighbor, in this case the bucket corresponding to R_2.

In contrast, the standard LSH algorithm generally uses one large radius (for example, R_5 in Figure 3-1) for all tables to maximize the number of candidates retrieved per table query. This is only done because it is computationally cheaper (outside of a privacy-preserving context) to perform brute-force comparison than it is to perform a hash table lookup.

3.4.7 The Comparison-free ANN Search Data Structure

Our new ANN data structure (presented in Figure 3-2) merges the above ideas to eliminate the brute-force search step present in the vanilla LSH-based data structure of Figure 2-2. The client can simply select any element from the first non-empty candidate set. This results in no comparisons when querying for the nearest neighbor which

will allow us to avoid *oblivious* comparisons when moving to the privacy-preserving protocol. Moreover, this new data structure can be further adapted to suppress database leakage, as we explain in the next section.

Comparison-free ANN search data structure

BUILD $(\mathcal{DB}, \mathcal{H}_1, \ldots, \mathcal{H}_L) \to (\mathcal{T}_1, \ldots, \mathcal{T}_L, \mathsf{h}_1, \ldots, \mathsf{h}_L)$.

Input:
- set of n vectors $\mathcal{DB} = \{\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}\}$,
- L hash families $\mathcal{H}_i$ each corresponding to an $R_L \le R_i \le R_L$ (Section 3.4.3).

Output:
- hash tables $\mathcal{T}_1, \ldots, \mathcal{T}_L$,
- and hash functions $\mathsf{h}_1, \ldots, \mathsf{h}_L$.

Procedure:
1: Sample random h_i from $\mathcal{H}_i$, for $i \in \{1, \ldots, L\}$.
2: Use h_i to build table $\mathcal{T}_i$ by hashing each vector $\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}$.
3: Output $\mathcal{T}_1, \ldots, \mathcal{T}_L$ and $\mathsf{h}_1, \ldots, \mathsf{h}_L$.

QUERY $(\mathcal{T}_1, \ldots, \mathcal{T}_L, \mathsf{h}_1, \ldots, \mathsf{h}_L, \boldsymbol{q}) \to \mathsf{ID}$.

Input:
- as in Figure 2-2.

Output:
- as in Figure 2-2.

Procedure:
1: Compute bucket key $l_i \leftarrow \mathsf{h}_i(\boldsymbol{q})$ and retrieve bucket $\mathcal{B}_{l_i}$ from $\mathcal{T}_i$.
2: $j \leftarrow$ min i with nonempty $\mathcal{B}_i$, or 0 if no such i exists.
3: **if** $j \neq 0$ **then** output any γ s.t. $\boldsymbol{v_\gamma} \in \mathcal{B}_j$; **else** output 0.

Figure 3-2: ANN search data structure with no direct distance comparisons between the query and the feature vectors in the candidate set.

3.4.8 Database Privacy and Suppressing Leakage

In this section we explain how the ideas of Section 3.4.3 apply to suppress database leakage. Our approach is a combination of three changes applied to QUERY in Figure 3-2. We recall that a simple strawman protocol achieving client privacy can be constructed by having the client privately retrieve colliding buckets through PIR [41]. We will assume that $L = \sqrt{n}$, and use these values interchangeably. However, we note that in practice the number of tables (L) can be smaller.

Step 1: Capping buckets. Because there is only one element to retrieve in expectation from the data structure of Figure 3-2, we can limit each hash bucket to only contain one vector. This ensures the size of the candidate set never exceeds $\sqrt{n}$.

Step 2: Hiding the vectors. Because the client selects the first non-zero ID from the candidate set using the data structure described in Figure 3-2, it does not need access to the vectors. As such, we can modify each hash table (BUILD; Figure 3-2) to only store the IDs of each vector. The client can still query the hash tables using PIR but now only obtains a candidate set of IDs (absent the vectors). If each vector in the database is d dimensional, then this simple change reduces leakage from $O(\sqrt{n} \cdot \log n \cdot d)$ bits to $O(\sqrt{n} \cdot \log n + d)$ bits (each ID is at least $\log n$ bits and $O(d)$ bits are leaked *implicitly* by the inference that the neighbor features are similar to the query).

Step 3: Hiding the candidate set. Compared to the ideal leakage of $O(\log n + d)$ bits, the leakage of $O(\sqrt{n} \cdot \log n + d)$ bits is far from optimal. We remove this extra leakage by designing a special "oblivious masking" transformation which hides all-but-one non-zero candidate ID from the client. From the masked candidate set $\tilde{\mathcal{C}}$, the client is only able to learn at most one ID that collided with its query. This further reduces leakage from $O(\sqrt{n} \cdot \log n + d)$ bits down to $O(\log n + d)$ bits (since only one ID is revealed), which matches the asymptotic leakage of the ideal functionality. We provide details on the oblivious masking transformation in Section 4.1.2, and a more formal leakage analysis in Chapter 5.

With these three leakage-suppressing steps, our protocol achieves close to optimal concrete database leakage per query. We show that asymptotically the leakage is actually optimal. Importantly, the leakage guarantees hold in the face of malicious clients that may deviate from protocol in an attempt to learn more about the database.

Chapter 4

The Protocol

In this chapter we introduce our privacy-preserving protocol for ANN search. In Section 4.1 we provide the necessary tools, both existing and new, which we use to instantiate the protocol in Section 4.2.

4.1 Building Blocks

4.1.1 Existing tool: Distributed Point Functions

A *point function* P_i is a function that evaluates to 1 on a single input i in its domain and evaluates to 0 on all other inputs $j \neq i$. A *distributed* point function (Definition 2) is a point function that is encoded into a pair of keys which are used to obtain a secret-shared evaluation of P_i on a given input j.

Definition 2 (Distributed Point Function (DPF) [30,31,56])**.** Fix any positive integer D. Let $\mathbb{F}$ be any finite field, and let λ be a security parameter. A DPF consists of two (possibly randomized) algorithms:

- $\mathsf{Gen}(1^\lambda, i \in \{1, \ldots, D\}) \to (k_A, k_B)$ takes as input the security parameter 1^λ and an index i. Outputs two evaluation keys k_A and k_B.
- $\mathsf{Eval}(k_\circ, j) \to v_j \in \mathbb{F}$ takes as input an evaluation key $k_\circ$ for $\circ \in \{A, B\}$ and an index $j \in \{1, \ldots, D\}$. Outputs a field element v_j.

These algorithms must satisfy correctness and privacy:

Correctness. A DPF is *correct* if for all pair of keys generated according to Gen,

$$\Pr\left[\ \mathsf{Eval}(k_A, j) + \mathsf{Eval}(k_B, j) = P_i(j) = \begin{cases} 1, & \text{if } j = i \\ 0, & \text{otherwise.} \end{cases}\ \right] = 1,$$

where the probability is over Gen. That is, the keys encode additive shares of the point function P_i.

Privacy. A DPF is *private* if each individual evaluation key output by Gen is pseudo-random (i.e., reveals nothing about i to a computationally bounded adversary). Formally, let K_b be the distribution of key $k_b \subset \{k_0, k_1\}$ where $(k_0,\ k_1)$ are sampled according to Gen. The DPF is considered private if there exists a probabilistic polynomial time (PPT) simulator Sim such that

$$K_b \approx_c \mathsf{Sim}(1^\lambda, 1^n, k_{1-b}).$$

In other words, the distribution of each DPF key output by Gen is efficiently simulatable and is therefore pseudo-random [56].

Application: symmetric PIR [55]. DPFs form the basis for efficient two-server PIR with optimal communication [30]. To see this, consider an ordered list S of n elements replicated on two servers. To retrieve the index i in S the client generates DPF keys and sends a share to each server. The servers evaluate the DPF on each index $j = 1, \ldots, n$ to obtain a secret-share of the point function evaluated on input j. Each server then multiplies the resulting secret-share by the corresponding entry in S at index j, and outputs the sum of all n component-wise products. That is, each server computes:

$$\sum_{j=1}^{n} \Big(S(j) \cdot \mathsf{Eval}(k, j) \Big) = S(i) \cdot [1] + \sum_{j \neq i}^{n} \Big(S(j) \cdot [0] \Big),$$

which results in a secret-share $[S(i)]$ of the ith value in S.

Because $P_i(j) = 1$ only when $i = j$, the resulting sum (computed in the field $\mathbb{F}$) is a

secret-share of $S(i)$. Summing the shares received from each server, the client recovers the desired entry in S. Observe that DPFs achieve *symmetric* PIR [55] (analogous to oblivious transfer [98]), which guarantees that the answer consists of at most one element in S.

Extension: PIR-by-keywords. The key size of modern two-server DPF constructions is $O(\log D)$ bits [30,31]. This allows for PIR with $O(\log D + \omega)$ communication (where ω is the size of elements in S). More generally, DPFs enable keyword-based PIR [41]. That is, for a key-value table consisting of n keys drawn from the domain $\{1, \ldots, D\}$, it is possible to use a DPF in a two-server setting to privately retrieve a value stored under key $i \in \{1, \ldots, D\}$, with only $O(\log D + \omega)$ communication and $O(n \cdot \omega)$ work on each server [30,47]. The latter follows from the ability to "lazily evaluate" DPFs on only the n keys present in the key-value table, rather than the full domain D. To see this, let $\mathcal{T}$ be a key-value map with set of keywords $S_{kw} \subseteq \{1, \ldots, D\}$, and suppose the DPF key k was generated according to $\mathsf{Gen}(1^\lambda, \tilde{\mathsf{w}})$ where $\tilde{\mathsf{w}} \in S_{kw}$. Each server computes:

$$\sum_{\mathsf{w} \in S_{kw}} \Big(\mathcal{T}(\mathsf{w}) \cdot \mathsf{Eval}(k, \mathsf{w}) \Big) = \mathcal{T}(\tilde{\mathsf{w}}) \cdot [1] + \sum_{\mathsf{w} \notin S_{kw}} \Big(\mathcal{T}(\mathsf{w}) \cdot [0] \Big),$$

which results in a secret share $[\mathcal{T}(\tilde{\mathsf{w}})]$ of the value at keyword $\tilde{\mathsf{w}}$ in $\mathcal{T}$.

4.1.2 New Tool: Oblivious Masking

The core of our leakage suppression technique (described at a high level in Section 3.4.8) hinges on the ability to reveal only the first non-zero value in a secret-shared vector. For example, given an input list of secret-shared field elements of the form

$$([0], [0], [0], [5], [0], [9], [15]) \in \mathbb{F}^7$$

we must transform it into a secret-shared vector of the form

$$([0], [0], [0], [5], [x], [x], [x]) \in \mathbb{F}^7,$$

where each x is a uniformly random value in $\mathbb{F}$. Because the vector is secret-shared (in some prime order field $\mathbb{F}$), this transformation must only involve affine operations: addition and scalar multiplication of shares [25,105]. The idea is to recursively compute a randomized sum, moving from left to right. For a secret-shared input vector $[\boldsymbol{v}] \in \mathbb{F}^t$, let $z \in \{1, \ldots, t\}$ be the index of the first nonzero element in $\boldsymbol{v}$. The randomized sums map each element $\boldsymbol{v}_i$ to 0 for $i < z$ and a uniformly random element in $\mathbb{F}$ for $i > z$. Crucially, this process does not affect the first non-zero element, $\boldsymbol{v}_z$. We will use this property to mask all-but-one result from a sequence of PIR query answers.

Algorithm 1: ObliviousMasking

Input:

- Secret-shared vector $[\boldsymbol{v}] \in \mathbb{F}^t$,
- Randomness source rand.

Output:

- Secret-shared vector $[\boldsymbol{y}] \in \mathbb{F}^t$.

Procedure:

1: **for** $i \in \{1, \ldots, t\}$**:**

1.1: Sample $r_i \xleftarrow{R} \mathbb{F}$ according to rand.

1.2: Set $[\boldsymbol{y}_i] \leftarrow [\boldsymbol{v}_i] + \boldsymbol{r}_i \cdot \left(\sum_{j=0}^{i-1} [\boldsymbol{v}_j] \right)$.

2: Output $[\boldsymbol{y}] \in \mathbb{F}^t$.

Claim 1. Let $\boldsymbol{v} \in \mathbb{F}^t$ be any vector and let $z \in \{1, \ldots, t\}$ be the first non-zero element of $\boldsymbol{v}$. Let $[\boldsymbol{v}]$ be an additive secret-sharing of $\boldsymbol{v}$. Algorithm 1 outputs a secret-shared vector $[\boldsymbol{y}]$ such that $\boldsymbol{y}_i = \boldsymbol{v}_i$ for $i \leq z$ and a uniformly random element of $\mathbb{F}$ for $i > z$.

Proof. The proof follows by examining the three possible cases for each value in $\boldsymbol{y}$ as a function of $\boldsymbol{v}$.

1. for $i < z$, $\boldsymbol{v}_i = 0$, so $\boldsymbol{y}_i = 0 + r_i \sum_{j=0}^{i-1} 0 = 0 \in \mathbb{F}$,
2. for $i = z$, $\boldsymbol{y}_z = \boldsymbol{v}_z + r_z \sum_{j=0}^{z-1} 0 = \boldsymbol{v}_z \in \mathbb{F}$,
3. for $i > z$, $\boldsymbol{y}_i = \boldsymbol{v}_i + r_i \sum_{j=0}^{z-1} 0 + r_i \boldsymbol{v}_z + r_i \sum_{j=z+1}^{i-1} \boldsymbol{v}_j \in \mathbb{F}$.

Case (1) ensures that all zeroes remain zeroes. Case (2) ensures that the first non-zero element is mapped to itself. Case (3) ensures that all subsequent elements are uniformly random in $\mathbb{F}$. To see why (3) holds, observe that $\boldsymbol{v}_z \neq 0$, so $r_i \cdot \boldsymbol{v}_z$ is a uniformly random

element of $\mathbb{F}$ given r_i is uniformly random. It then follows that the sum is uniformly random in $\mathbb{F}$. Finally, correctness of the computation over secret-shares follows from all operations performed above being linear (additions and scalar multiplications) over the input secret-share of $\boldsymbol{v}$ [25]. ■

Remark 3. The common randomness source is required to ensure that Algorithm 1 can be evaluated by different parties holding shares of $\boldsymbol{v}$ (the input vector). Specifically, scalar multiplication of secret-shared values requires the scalar to be known to all parties holding the shares [25]. In practice, rand can be easily instantiated by agreeing on a short pseudo-random generator seed among the parties. Each party can then sample r_i using the pseudo-random generator, which will ensure that the same r_i is sampled by all parties.

4.2 Putting Things Together

The full protocol is presented in Protocol 1 and uses the DPF and oblivious masking building-blocks described in Section 4.1. We briefly describe each step of the protocol.

Setup. The public parameters consist of the number of hash tables (L) and a list of L randomly sampled hash functions, in accordance with the data structure of Figure 3-2 and Proposition 1. The servers construct L hash tables using the hash functions from the public parameters. Only the IDs (i.e., indices) of the input vectors are stored in the hash table; the vectors are discarded (see Section 3.4.8).

Step 1. The client hashes its query vector $\boldsymbol{q}$ using the LSH functions in the public parameters. Each resulting hash is used as the keyword in DPF.Gen to generate a DPF key (see PIR-by-keywords extension described in Section 4.1). The generated DPF keys are distributed by the client to each server.

Step 2. Each server uses the L DPF keys it receives from the client to retrieve a secret-share of a bucket in each of the L hash tables using PIR-by-keywords. The result is a list $\mathcal{C}$ of L secret-shared buckets containing either a candidate ID (or zero if the bucket was empty). Each server applies the oblivious masking transformation

Protocol 1: Private Similarity Search

Public input: LSH family $\mathcal{H}$, number of hash tables L, and LSH functions $\mathsf{h}_1, \ldots, \mathsf{h}_L$ sampled i.i.d. from $\mathcal{H}_i$.
Servers input: Database of vectors $(\boldsymbol{v_1}, \ldots, \boldsymbol{v_n})$.
Client input: Query vector $\boldsymbol{q}$.

Setup (one-time server-side pre-processing)
On each server
1: **for** $i \in \{1, \ldots, L\}$**:**
1.1: Construct hash table $\mathcal{T}_i$ by storing j in bucket with key $l_j := \mathsf{h}_i(\boldsymbol{v_j})$ for all $j \in n$.
1.2: Truncate each bucket in $\mathcal{T}_i$ have at most one value; discarding values at random.
2: Agree on common randomness source rand. // e.g., a PRG seed.

Protocol
Step 1: (On the client)
1: **for** $i \in \{1, \ldots, L\}$**:**
1: $l_i \leftarrow \mathsf{h}_i(\boldsymbol{q})$. // get hash key for table $\mathcal{T}_i$.
2: $(k_A^{(i)}, k_B^{(i)}) \leftarrow \mathsf{Gen}(l_i)$. // DPF keys for $\mathcal{T}_i$ query.
2: $\boldsymbol{k}_A \leftarrow (k_A^{(1)}, \ldots, k_A^{(L)})$ and $\boldsymbol{k}_B \leftarrow (k_B^{(1)}, \ldots, k_B^{(L)})$.
3: Send $\boldsymbol{k}_A$ and $\boldsymbol{k}_B$ to servers A and B, respectively.

Step 2: (On each server)
1: Parse $\boldsymbol{k} = (k^{(1)}, \ldots, k^{(L)})$. // set of DPF keys.
2: **for** $i \in \{1, \ldots, L\}$**:**
2.1: $n_i \leftarrow$ number of bucket keys $(l_1, \ldots, l_{n_i})$ in hash table $\mathcal{T}_i$.
2.2: $[a_i] \leftarrow \sum_{j=1}^{n_i} \mathcal{T}_i(l_j) \cdot \mathsf{Eval}(k^{(j)}, l_j)$. // PIR-by-keywords.
3: $\mathcal{C} \leftarrow ([a_1], \ldots, [a_L])$.
4: $\tilde{\mathcal{C}} \leftarrow \textsc{ObliviousMasking}(\mathcal{C}, \mathsf{rand})$. // Algorithm 1.

Step 3: (On the client)
1: Receive $\tilde{\mathcal{C}}_A$ and $\tilde{\mathcal{C}}_B$ from servers A and B, respectively.
2: **for** $i \in \{1, \ldots, L\}$**:**
2.1: $j_i \leftarrow \tilde{\mathcal{C}}_A(i) + \tilde{\mathcal{C}}_B(i)$. // recover secret-shared ID.
3: Output $j_i \neq 0$ for smallest i, or 0 if all j_i are zero.

(Algorithm 1) to $\mathcal{C}$ and obtain the masked secret-shared list $\tilde{\mathcal{C}}$ as output. Each server sends its share of $\tilde{\mathcal{C}}$ to the client in response.

Step 3. From the received secret-shares $(\tilde{\mathcal{C}}_A, \tilde{\mathcal{C}}_B)$, the client recovers the list $\tilde{\mathcal{C}}$ (in

cleartext) by computing $\tilde{\mathcal{C}} = \tilde{\mathcal{C}}_A + \tilde{\mathcal{C}}_B \in \mathbb{F}^L$.[1] The transformation applied by the servers in Step 2 ensures that the client learns at most *one* non-zero candidate from the original $\mathcal{C}$. Specifically, by Claim 1, only the first non-zero value of $\mathcal{C}$ will be non-random in $\tilde{\mathcal{C}}$. The first non-zero value, if present, is output to the client.

Querying for k-nearest Neighbors. The canonical ANN search problem involves finding *one* nearest neighbor [13,46,57,84], which easily extends to k-nearest neighbors [57]. Intuitively, the idea is to treat each "group" of k values as a single result, which slots in with Protocol 1. To wit, we can change the bucket size to k and tune the parameters to get k elements per bucket, in expectation. This may result in partially filled buckets. As such, buckets with fewer than k non-zero values are discarded. Alternatively, depending on the correctness requirements, all buckets with fewer than $k - a$ values can be discarded, for some $a < k$. With this simple change, the client can now retrieve a non-zero bucket (containing k values) after the oblivious masking step. This results in the output of the k-nearest neighbors.

Concrete efficiency. While Protocol 1 satisfies our asymptotic efficiency requirements, it requires a few more optimizations to achieve concrete efficiency. Specifically, we show how to optimize query processing time on the server using hash table multi-probing [84,95] and a new probabilistic batch-processing technique [14,15,68] for PIR.

4.3 Optimizations

We now turn our attention to making the protocol concretely efficient. The primary bottleneck in Protocol 1 comes from the processing time needed to retrieve L buckets using the provided DPF keys. While this processing time does not involve the client, it can lead to high latency in answering queries, as each query requires $O(n)$ server-side work per hash table.

The seminal work of Ishai et al. [69] provides a way to amortize the processing cost of performing multiple PIR queries using *batch codes*. Batch-PIR [69], realized batch codes, computes the answer to k simultaneous PIR queries at an amortized

[1] Additive secret-share recovery follows from correctness property of DPFs; see Definition 2.

processing cost. While the idea of batching queries is appealing, it does not directly apply to Protocol 1. The reason is that batch-PIR only works when retrieving multiple elements from the *same* table; not when querying different tables. To align with the requirements of batch-PIR, we use a technique known as LSH multi-probing [84,95]. Multi-probing retrieves multiple keys in each hash table and reduces the total number of hash tables in the ANN data structure. Then, we can use batching techniques to retrieve all multi-probes from each table efficiently. However, we find that the traditional way of performing batch-PIR through coding schemes results in too much overhead in probabilistic settings. To remedy this, we develop a new tool we call *partial batch retrieval*. We show that combining these two techniques results in several orders of magnitude faster processing time.

4.3.1 LSH Multi-probing

The idea behind LSH multi-probing is that if the bucket to which the query hashes to in a table is empty, then it is likely that "adjacent" buckets in the table contain the collision [84,95]. To exploit this observation, each hash table is probed on ℓ keys which "surround" the query. The surrounding keys are computed by hashing a perturbed version of the query vector, leading to a higher likelihood of finding a candidate, if it exists. In turn, this reduces the number of hash tables (L) required (the original motivation of multi-probing [84]). The appealing property to our use case, however, is that multi-probing allows us to apply batch-PIR techniques and heavily amortize processing time when retrieving candidates. The only change to Protocol 1 is that now the client retrieves up to ℓ candidates from each hash table, instead of just one candidate. This reduces the number of hash tables (L) required by a factor of ℓ. Meanwhile, answering a batch of ℓ queries can be amortized to approximately the cost of answering one query, which we show next.

4.3.2 Partial Batch Retrieval

Batch codes [68] have the strict requirement that *any* combination of ℓ elements should be retrievable with perfect probability. With Partial Batch Retrieval (PBR) (inspired by *probabilistic* batch codes [14,15]), we relax this requirement: we only care about retrieving some *fraction* of the ℓ elements, with high probability.

The main idea. A simple PBR can be realized by dividing the elements into $m \geq \ell$ partitions at random. If all ℓ elements a client would like to retrieve fall into a unique partition, then it suffices for the client to issue m PIR queries, with each query retrieving an element from a partition. The total server processing time to compute PIR answers remains the same. To see this, observe that for a database of n elements, each partition contains n/m elements. The total work to answer m PIR queries, each computed over a set of n/m elements, is then amortized to $O(n)$.

However, the success of this PBR scheme hinges on elements falling into unique partitions. How many elements can we expect to retrieve in practice?

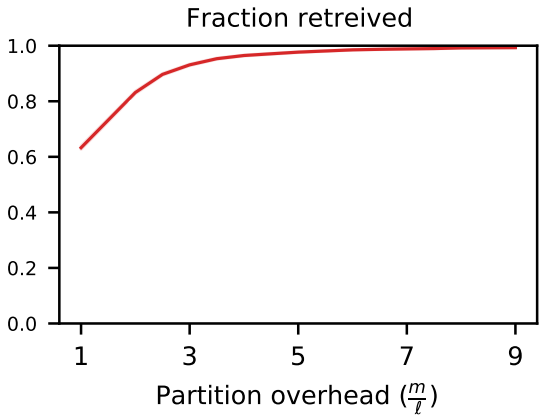

Figure 4-1: Expected fraction of ℓ elements that can be retrieved from m partitions using partial batch retrieval.

Abstractly, the fraction of retrievable elements can be modeled by the classic "balls and bins" problem [87], where ℓ balls are tossed into m bins uniformly at random. If all ℓ balls fall into unique bins, then the number of full bins is ℓ. Fewer than ℓ full bins corresponds to a collision in a bin (i.e., partition) and only one element in each partition can be retrieved. Let X_i be the indicator random variable where $X_i = 1$ if a bin is full. Then,

$$\Pr[X_i = 1] = 1 - \left(1 - \frac{1}{m}\right)^{\ell} > 1 - e^{-\ell/m},$$

which implies that $\frac{m}{\ell} \cdot (1 - e^{-\ell/m})$ of the ℓ elements are simultaneously "retrievable" from

Batch-PIR scheme	Replication Factor	Partitions	Fraction Retrieved
Naive	1	1	$\frac{1}{\ell}$
Subcube batch code [68]	$(\frac{3}{2})^{\ell}$	$3^{\log \ell}$	1
Probabilistic batch code [15]	3	1.5ℓ	$> 1 - 2^{-20}$
Partial batch retrieval	1	m	$\frac{m}{\ell} \cdot (1 - e^{-\ell/m})$

Table 4.1: Replication and partitioning costs of existing batching schemes. Replication increases server-side processing by the same factor but influences communication by a sublinear factor. Communication increases linearly with the number of partitions for all schemes.

the $m \geq \ell$ partitions. In the case that $m = \ell$, we can expect to retrieve approximately 63% of the elements. With $m > \ell$, we can increase the probability of retrieving all elements at the cost of also increasing communication by a factor of $\frac{m}{\ell}$. We plot the expected fraction of retrievable elements in Figure 4-1.

Comparison to batch codes. It is natural to ask how PBRs compare to batch codes. The main difference is with respect to the guarantees provided by both tools. Batch codes (and their probabilistic variants [14,15]) aim to guarantee retrieval of *all* ℓ elements. Doing so comes at an efficiency cost: batch codes *replicate* data which increases both processing time and bandwidth. With PBRs, we avoid the need for replicating data, keeping the total *processing* cost fixed while only modestly increasing bandwidth (see Table 4.1 and Figure 4-2 for a comparison to batch codes). However, this comes at a different cost: PBRs only guarantee *partial* retrieval of the ℓ elements. We find that for certain applications, such as Protocol 1, partial retrieval in favor of decreased processing time is a desirable trade-off, given the already probabilistic guarantees of LSH.

Application to Protocol 1. We apply the above PBR scheme to retrieve a fraction of multi-probe hash keys from each hash table in Protocol 1. Our main observation is that failing to retrieve specific probes is not a total failure. It is equivalent to not choosing that particular multi-probe, which could have already happened due to the probabilistic nature of LSH. Because each multi-probe hash key is uniformly distributed from universal hashing (see Remark 1), each hash key is equally likely to be selected, making it possible to directly apply our PBR scheme. The use of a PBR

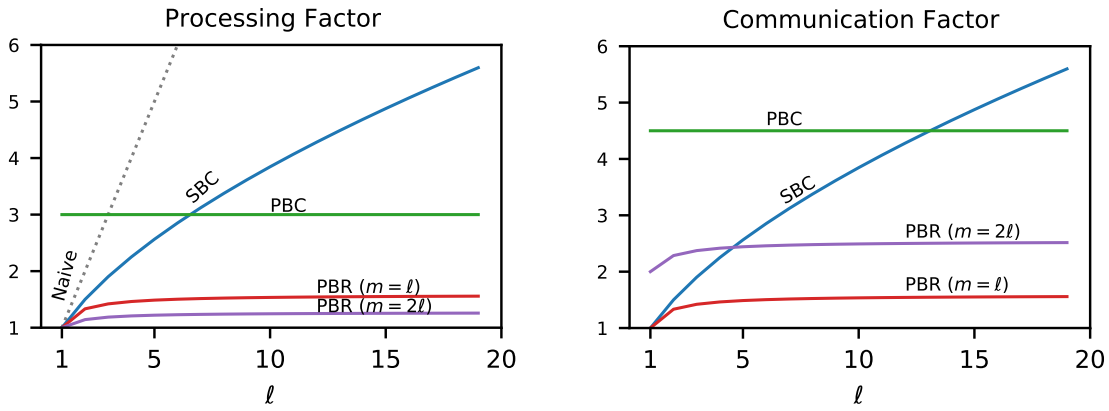

Figure 4-2: Concrete overhead factor, in terms of communication and processing time, for retrieving ℓ elements using our partial batch retrieval (PBR; Section 4.3.2), subcube batch codes (SBCs [68]), and probabilistic batch codes (PBCs [15]).

makes Protocol 1 both concretely and asymptotically efficient. The number of hash tables L decreases by a factor proportional to ℓ, amortizing the processing cost for both servers. Setting $m = \ell = L/a$, for some constant a, results in total amortized processing time that is $O(n)$ (as referenced in Section 4.4), which matches the PIR processing lower bound [40]. Because all batch codes rely on replicating data [14,15,68], it is more efficient to construct a new hash table (and querying a fresh set of multi-probes) using our PBR scheme than it is trying to ensure that all probes are successful.

4.4 Asymptotic Efficiency Analysis

We analyze the communication and computation costs individually (summarized in Table 4.2). To derive our asymptotic guarantees, we follow the analysis of Andoni and Indyk [8] and our analysis of Proposition 1 in Section 3.4.4. Specifically, we recall the definition of ρ. In our analysis, we assume the approximation factor $c = 2$. This is required for asymptotically sublinear communication, as discussed in Section 3.4.4.

Communication	Server Work	Client Work	Rounds
$\sqrt{n}$	n	$\sqrt{n}$	1

Table 4.2: Asymptotic efficiency of Protocol 1; constant and log factors suppressed for readability. We assume $L \approx \sqrt{n}$ and $c = 2$; see Section 2.2.

Communication. The protocol communication consists of the L DPF keys, each of which retrieves one candidate from a table. The DPF key size is $O(\log n)$ [30,31]. Each masked ID returned by the protocol is of size $\log n$. Therefore, when $L \approx \sqrt{n}$, the total communication is $O(\sqrt{n} \cdot \log n)$. In the multi-probing case, the total number of *candidates* retrieved is still $\sqrt{n}$, even though the number of hash tables (L) is smaller.

Computation. For each server, computing the answer to the PIR query requires evaluating the DPF on $O(n)$ hash keys, per hash table. Therefore, the total server work is naïvely $O(L \cdot n)$. Using partial batch retrieval described in Section 4.3.2, we can amortize this processing cost to $O(n)$ by retrieving multiple candidates from each hash table simultaneously. Specifically, we can set $L = \ell$, for some constant $\ell \geq 1$ and retrieve $\sqrt{n}/\ell$ multi-probes from each hash table. This results in an amortized computation cost of $O(n)$ on each server. In practice, we will set L so as to have sufficiently fine-grained radix-sort subdivisions; see Section 6.2.1. After fixing L, we set $\ell = \sqrt{n}/L$ so as to retrieve $\sqrt{n}$ candidates in total.

Oblivious masking, which is performed over the candidate set, requires $O(\sqrt{n})$ additional work per server. For the client, the computational cost is proportional to the query response size, i.e., $O(\sqrt{n})$. This follows from the client having to receive and recover the candidate set in order to find the first non-zero element.

Rounds. The protocol requires only a single round of communication between the client and each server. The servers do not communicate when answering client queries.

Storage. The storage overhead on the client is simply the query $\boldsymbol{q}$ which requires $O(d)$ bits (assuming the feature vector consists of constant entries). On the server, the storage overhead is $O(L \cdot n \log n)$ for storing all hash tables.

Chapter 5

Security and Leakage Analysis

In this section, we analyze the security of Protocol 1 with respect to client privacy and server leakage. While client privacy is conceptually simple and follows directly from the privacy property of DPFs (Definition 2), the leakage analysis of the database is more involved.

5.1 Client Privacy

Claim 2 (Query Privacy)**.** For all Probabilistic Polynomial Time (PPT) adversaries $\mathcal{A}$ corrupting server $\circ$ for $\circ \in \{\mathrm{A}, \mathrm{B}\}$, Protocol 1 guarantees that $\mathcal{A}$ learns no information on the client's query, even when deviating from protocol. This holds even when the query-batching optimization (Section 4.3) is applied to Protocol 1.

Proof. The proof follows from a simulation-based indistinguishability argument. We say that the protocol is *query private* if there exists a PPT simulator Sim such that

$$\mathsf{Sim}(\mathcal{DB}) \approx_c \mathsf{view}(\mathcal{DB}, \boldsymbol{k}_\circ),$$

where view is the view of $\mathcal{A}$ from an execution of Protocol 1 with the client. $\mathsf{Sim}(\mathcal{DB})$ is trivially constructed by invoking the DPF simulator for each DPF key present in the vector $\boldsymbol{k}_\circ$ provided to $\mathcal{A}$ [34]. The client does not send any other information to the servers apart from the DPF keys which are used to query the hash tables. Therefore,

query privacy depends only on the privacy property of DPFs and follows immediately.

With multi-probing and batching (optimizations described in Section 4.3), the client retrieves ℓ keys from each table, where the key space is partitioned into m uniformly random subsets (see Section 4.3.2). This results in m DPF keys sent to each server per hash table. Because the client sends a DPF key for each partition (even if no bucket is retrieved from that partition), no information is revealed to $\mathcal{A}$. Therefore, query privacy follows by the same argument. See also Ishai et al. [68]. ■

5.2 Database Privacy and Leakage

We now turn to formalizing the database leakage incurred when answering a client query. We must be careful to define database leakage in a meaningful way. As observed by Zuber and Sirdey [121]: *"[H]aving access to the result of the classification leads to some (useful) leakage on the raw database."* As such, we must start from a *baseline leakage* given that the client obtains information as output: the ANN ID relative to the query $\boldsymbol{q}$. In our setting, the client learns strictly more than just the ID of the ANN, which we must quantify as additional leakage. We emphasize that this leakage is with respect to the server's database, **not** the client's query.

Theorem 1 (Quantifying Ideal Leakage)**.** Fix quantities D_{max}, D_{min}, and Δ as defined in Section 3.2. The ideal leakage for an instance of approximate nearest neighbor search as instantiated in Functionality 1 is captured by $O(d+\log n)$ bits of information per query, where d is the intrinsic dimensionality of the vector space and n is the number of vectors in the database.

Proof. We start by considering the ℓ_∞-norm and induced distance metric Δ. The ℓ_∞-norm is the absolute value of the maximum coordinate: $\ell_\infty||x|| = \max_{\boldsymbol{x}_i} |\boldsymbol{x}_i|$. The induced distance metric is $\Delta(\boldsymbol{x}, \boldsymbol{y}) = \ell_\infty||\boldsymbol{x} - \boldsymbol{y}||$. We first prove the ideal leakage for the ℓ_∞-norm, as it is easier to intuit. We then show how to extend the proof to any ℓ_p-norm induced metric, which includes Euclidean distance. Other common distance metrics, such as angular distance and hamming distance, can be embedded into Euclidean space [38] [18].

Recall that for any query $\boldsymbol{q}$ that will return an ID corresponding to vector $\boldsymbol{v}$, we have that $\Delta(\boldsymbol{v}, \boldsymbol{q}) \leq R_{\mathsf{max}} < D_{\mathsf{max}}$. The ideal functionality implicitly reveals that there exists a point $\boldsymbol{v}$ such that $\Delta(\boldsymbol{v}, \boldsymbol{q}) \leq R_{\mathsf{max}}$. In the ℓ_∞ metric, this implies that $\boldsymbol{v}$ is within a cube of side length $2R_{\mathsf{max}}$ centered at $\boldsymbol{q}$. The output implicitly reveals that $\boldsymbol{v}$ is in this cube. Considering each coordinate of $\boldsymbol{v}$, the number of possible values each coordinate can take diminishes from $2D_{\mathsf{max}}$ to $2R_{\mathsf{max}}$. As a result, the information revealed is $\log(D_{\mathsf{max}}/R_{\mathsf{max}})$ bits. With the ℓ_∞ metric, this argument can be repeated individually for each coordinate, making the total leakage $d\log(D_{\mathsf{max}}/R_{\mathsf{max}})$ bits.

For any ℓ_p-norm, the above argument holds by considering similar shapes and their relative volumes. In the Euclidean metric $\Delta(\boldsymbol{v}, \boldsymbol{q}) \leq D_{\mathsf{max}}$ defines a ball of radius D_{max}. The ideal functionality reveals that $\boldsymbol{v}$ is in a ball of radius R_{max} centered at $\boldsymbol{q}$. The ratio of the volumes of these balls is $D^d_{\mathsf{max}}/R^d_{\mathsf{max}}$, making the leakage $\log(D^d_{\mathsf{max}}/R^d_{\mathsf{max}}) = d\log(D_{\mathsf{max}}/R_{\mathsf{max}})$ bits, as in the ℓ_∞ metric. ■

Intuitively, this leakage corresponds to the fact that the query vector serves as an approximation for the feature vector of the nearest neighbor, by definition of the problem. The precision of this approximation is limited to $\log(D_{\mathsf{max}}/R_{\mathsf{max}})$.

We now formalize the leakage per query in Protocol 1 and show that it is no worse than $O(d + \log n)$ bits, which matches the leakage of the ideal functionality up to a constant factor.

Claim 3 (Asymptotic Leakage of Protocol 1)**.** Let L be the number of hash tables used to instantiate Protocol 1. Then, the leakage of Protocol 1 is $O(d + \log n)$ bits, matching the ideal functionality.

Proof. Consider the leakage of an infinite number of queries answered through Protocol 1. Fix any $\boldsymbol{v_j} \in \mathcal{DB}$. The total information revealed on $\boldsymbol{v_j}$ is never more than the set $S_j = \{l_1, \ldots, l_L \mid l_i = \mathsf{h}_i(\boldsymbol{v_j})\}$. That is, the set of all L LSH digests of $\boldsymbol{v_j}$. The maximum information revealed is thus the set S_j for all n feature vectors $\boldsymbol{v_j} \in \mathcal{DB}$.

For an individual query, the oblivious masking transformation (Section 4.1.2) guarantees that at most one element of S—a hash corresponding to a feature vector $\boldsymbol{v} \in \mathcal{DB}$—is leaked. Consider the worst-case (least-hiding) LSH function possible. This

is not even a locality-sensitive hash function but instead simply the identity function $\mathsf{h}(\boldsymbol{x}) = \boldsymbol{x}$. It follows that the worst-case implicit query leakage is then $O(d)$ bits, because $l_i = \boldsymbol{v_j}$ for all $i \in \{1, \ldots, L\}$). This is in addition to the explicit $\log n$ bits revealed by the vector ID. Therefore, the total leakage is $O(d + \log n)$, which matches the leakage of the ideal functionality in Theorem 1. ■

Claim 4. A client deviating from Protocol 1 cannot learn more information on $\mathcal{DB}$ than an honest client following protocol.

Proof. Fix S_j (the set of all hashes) as defined in the proof of Claim 3. By the guarantees of the oblivious masking transformation (Claim 1), even a malicious client can only obtain one element of S_j (for some $j \in \{1, \ldots, L\}$). All other elements are uniformly random or zero. Even if the client does not follow the protocol (such as sending random hashes that don't correspond to any one query), the leakage is capped at a single element of S_j. ■

5.2.1 Bounding Concrete Leakage

From the proof of Claim 3, we see that the asymptotic leakage consists of at most one LSH digest, even when the client is acting maliciously (Claim 4). In the worst case, this digest corresponds to a full feature vector, leading to the asymptotic bound. However, concretely, we would like to analyze *how much worse* Protocol 1 is in comparison to Functionality 1. We answer this in Claim 5. We show that Protocol 1, when instantiated for Euclidean distance using the Leech lattice LSH (Section 3.4.5), leaks a more precise approximation compared to Functionality 1. Intuitively, this extra leakage comes from the client learning which radix bucket the nearest neighbor to the query is located in. To analyze the concrete leakage, we must first establish some technical groundwork pertaining to the Leech-lattice LSH we use for Euclidean (and Angular) distance. We do so in Lemma 1, where we will show that a Leech-lattice LSH digest is inherently less precise than the corresponding ideal ball in Euclidean space. Specifically, we show that the ideal ball of radius R has a smaller volume compared to the region of space represented by an LSH digest.

Lemma 1. *The* $(1, 2, 0.0097459, 0.0000156)$*-sensitive Leech lattice LSH [8] has volume at least* $\frac{V_B}{0.77}$*, where* V_B *is the volume of the* 24*-dimensional unit ball.*

Proof. Our proof relies on the following fact about the Leech lattice.

> **Fact 1:** The Leech lattice is a 24 dimensional object with a sphere packing density of $\frac{\pi^{12}}{12!}$ [43], where the density is defined as the fraction of space covered by balls centered at the lattice points.

By Fact 1, we have that the ratio of the volume of the unit ball to that of the lattice cell is given by the density. A small calculation shows that the lattice cell then has volume 1. We ask what is the minimum radius for a 24-dimensional ball such that the ball has volume 1. We find that the answer is

$$\sqrt[24]{\frac{12!}{\pi^{12}}} \approx 1.29.$$

The inverse of this quantity is 0.77, which proves the lemma. The values of $p_1 = 0.0097459$ and $p_2 = 0.0000156$ for $c = 2$ are from [8, Table 1]. ∎

Remark 4. In Lemma 1 we describe the $(1, 2, 0.0097459, 0.0000156)$-sensitive Leech lattice LSH for Euclidean space as described by Andoni and Indyk [8]. We note that this LSH can be used for any R and cR (while also preserving the same p_1 and p_2), by simply scaling the input vectors accordingly. See Andoni and Indyk [8] for a more detailed explanation.

Using Lemma 1, we can bound the concrete leakage of Protocol 1 when instantiated with the Leech-lattice LSH. This allows us to empirically bound the concrete leakage for the datasets we use in our evaluation (Chapter 6). We compute a precise leakage bound in Claim 5.

Claim 5 (Concrete Leakage of Protocol 1)**.** Fix R_{min} and R_{max} as defined in both Figure 3-2 and Functionality 1, where $D_{\mathsf{min}} < R_{\mathsf{min}} \leq R_{\mathsf{max}} < D_{\mathsf{max}}$. Protocol 1, when instantiated with the Leech-lattice based LSH of Andoni and Indyk [8] leaks at most a multiplicative factor of $0.77 \cdot \frac{R_{\mathsf{max}}}{R_{\mathsf{min}}}$ more information compared to Functionality 1.

Proof. We recall the argument in the proof of Claim 3. Fix any $\boldsymbol{v_j} \in \mathcal{DB}$ and S_j as in the proof of Claim 3. We show that S_j is an upper bound on what can be revealed on $\boldsymbol{v_j}$ through Protocol 1. We now examine how much faster this set can be leaked with queries issued to Protocol 1 and contrast it to Functionality 1.

Define an ideal oracle $\mathcal{O}(\mathcal{DB}, R_i, \cdot)$, which given a query $\boldsymbol{q}$, outputs the ID of any vector in $\mathcal{DB}$ within distance R_i of $\boldsymbol{q}$, if such a vector exists. Observe that Functionality 1 is modeled by $\mathcal{O}(\mathcal{DB}, R_{\mathsf{max}}, \cdot)$.

Suppose that the client obtains a query answer and learns $l_i \in S_j$. Note that this leakage is less than (or equal to) that of learning $l_1 \in S_j$. That is, the hash of the same vector but on the smallest radius. It is easy to see that l_1 is at least as precise (contains as much information) as $l_i \in S_j$, $i > 1$, due to the increasing radii of the radix buckets, as explained in Section 3.4.4.

Next, by Lemma 1, we have that the information revealed by $l_1 = \mathsf{h}_1(\boldsymbol{q})$ is less than or equal to the information revealed by $\mathcal{O}(\mathcal{DB}, R_1, \boldsymbol{q})$. Indeed, as was shown in Lemma 1, l_i is 0.77 less precise compared to $\mathcal{O}(\mathcal{DB}, R_i, \boldsymbol{q})$, for any i.

Therefore, we have that the ratio in precision between l_1 and $\mathcal{O}(\mathcal{DB}, R_{\mathsf{max}}, \cdot)$, that is, the ratio between the precision of the smallest radix bucket and the ideal functionality, is bounded by 0.77 times the ratio of precision between the ideal oracles $\mathcal{O}(\mathcal{DB}, R_{\mathsf{min}}, \cdot)$ and $\mathcal{O}(\mathcal{DB}, R_{\mathsf{max}}, \cdot)$, respectively. Because the latter ratio is simply $\frac{R_{\mathsf{max}}}{R_{\mathsf{min}}}$, we get that Protocol 1 leaks at most a multiplicative factor of $0.77 \cdot \frac{R_{\mathsf{max}}}{R_{\mathsf{min}}}$ more compared to the ideal leakage of Functionality 1, derived in Theorem 1. ∎

We empirically compare the concrete leakage of Protocol 1 to the ideal leakage in Section 5.2.2 on real world data. We do so by finding values for R_{min} and R_{max} as a function of L (see Section 6.2.1 for how this is done) and applying Claim 5.

Corollary 1. The leakage of k queries to Protocol 1 is bounded by a multiplicative factor of $0.77 \cdot \frac{R_{\mathsf{max}}}{R_{\mathsf{min}}}$ more than the leakage of k queries to Functionality 1.

Proof. We only need to consider the relative leakage between k successive queries to $\mathcal{O}(\mathcal{DB}, R_{\mathsf{min}}, \cdot)$ and $\mathcal{O}(\mathcal{DB}, R_{\mathsf{max}}, \cdot)$, resulting in at most a factor of k more leakage. Another attack one might consider is to learn more information across multiple queries.

While the information given by k hashes for the same vector is more specific (e.g., the set of points that have both hashes is smaller), the same effect occurs with queries to the ideal oracle. If two queries spaced by some offset return the same nearest neighbor, then that neighbor must be in the intersection of the regions of radius R_{max} centered at each query. The leakage factor captures the extra precision in our case. It also might be the case that the malicious client is able to pick queries by exploiting hashes that return zeroes. However, this is bounded by obtaining a new element of S_j (the set of all LSH digests for a vector $\boldsymbol{v_j}$; see proof of Claim 3) each time, which is captured in our concrete leakage bound. ■

5.2.2 Empirical Leakage Calculation

We chart the leakage factor in Table 5.1 for the real world datasets described in Table 6.1. For each dataset, different radii are required in order to separate near and nearest neighbors using the radix approach of Section 3.4.3. It is this extra precision that is captured in the leakage factor. We note that this leakage is a worst-case bound on the actual concrete leakage. Specifically, this leakage assumes that *all* points collide at the smallest radius R_{min}, when in practice the points will collide across buckets defined by radii between R_{min} and R_{max}.

Dataset	$L = 10$	$L = 30$	$L = 50$
DEEP1B	$0.77 \cdot \frac{0.79}{0.31} = 1.97\times$	$0.77 \cdot \frac{0.86}{0.23} = 2.79\times$	$0.77 \cdot \frac{0.89}{0.21} = 3.29\times$
MNIST	$0.77 \cdot \frac{1572}{559} = 2.16\times$	$0.77 \cdot \frac{1720}{410} = 3.23\times$	$0.77 \cdot \frac{1781}{349} = 3.92\times$
GIST	$0.77 \cdot \frac{1.04}{0.25} = 3.25\times$	$0.77 \cdot \frac{1.15}{0.13} = 6.86\times$	$0.77 \cdot \frac{1.20}{0.81} = 11.32\times$
SIFT	$0.77 \cdot \frac{246}{54} = 3.52\times$	$0.77 \cdot \frac{275}{25.7} = 8.24\times$	$0.77 \cdot \frac{14.06}{1.20} = 15.7\times$

Table 5.1: Relative leakage $\frac{R_{\mathsf{max}}}{R_{\mathsf{min}}}$ versus ideal following Claim 3, evaluated on four real datasets. Leakage increases roughly proportional with the number of tables (L) given that each additional table introduces more precision in the resulting answer (assuming additional tables are used to increase accuracy). See Section 6.2.1 for more details. The 0.77 factor comes from the imprecision of the LSH computed in Lemma 1.

Chapter 6

Evaluation

We now turn to describing our implementation and empirical evaluation of Protocol 1. The goal of this section is to answer the following questions:

- What are the parameters needed to obtain high accuracy in practice using the data structure of Figure 3-2?
- What is the concrete performance of Protocol 1 when used for ANN search on real data?
- How does Protocol 1 compare to the state-of-the-art approach for private similarity search?

6.1 Implementation and Environment

We implement Protocol 1 in approximately 4,000 lines of code. Our implementation is written in Go v1.16. with performance-critical components written in C. The code is open source and available online [2]. Our DPF implementation follows Boyle et al. [31] and is partially based on open-source libraries [47,50]. Our implementation uses AES as a pseudo-random generator which exploits the AES-NI instruction for hardware-accelerated operations. We use the GMP library [52] for fast modular arithmetic.

6.1.1 Environment

We deploy our implementation on Amazon Elastic Cloud Compute (EC2) for our experiments. We geographically locate the servers on the east coast of the United States while the client is located on the west coast. We measure average ping time of 79 ms between the client and servers. Each server runs on a `c5.9xlarge` Amazon Linux virtual machine. Each server is equipped with Intel Xeon Platinum 8000 CPU (36 vCPUs) and 72 GB of RAM. We use a small `t2.micro` (1 vCPU; 1 GB of RAM) VM for the client (specifications comparable to a low-end smartphone).

6.1.2 Implementation of the Leech-lattice LSH

We find the closest Leech lattice point to a specified point (e.g., the query) using the decoder described by Conway and Sloane [42]. The coordinates of the lattice point are then mapped to the DPF domain size (we set $D = 2^{64}$; Definition 2) using a universal hash. Since the Leech lattice is a 24-dimensional object, the first step of Andoni and Indyk [8, Appendix B] is dimensionality reduction [71]. We randomly project $d/2$ of the coordinates onto each lattice before concatenating the hashes (we use $k = 2$; see Proposition 1). Many locality sensitive hashing algorithms have efficient methods for multi-probing [84]. For lattice based hashes, we choose the multi-probes from the set of closest lattice points, as these correspond to unique hash values. Indeed, the client can advantageously select the closer lattice points when retrieving candidates through PBR (Section 4.3.2), letting the retrieval failures correspond to probes that are further away. This results in a small increase in multi-probe accuracy.

6.2 Datasets and Parameters

We evaluate our implementation on four real-world datasets obtained from the ANN benchmarks repository [1,18]. See Table 6.1 for summary of dataset characteristics.

DEEP1B (Deep billion-scale indexing) [21] contains one billion image embeddings (feature vectors) resulting from the last fully-connected layer of the pretrained

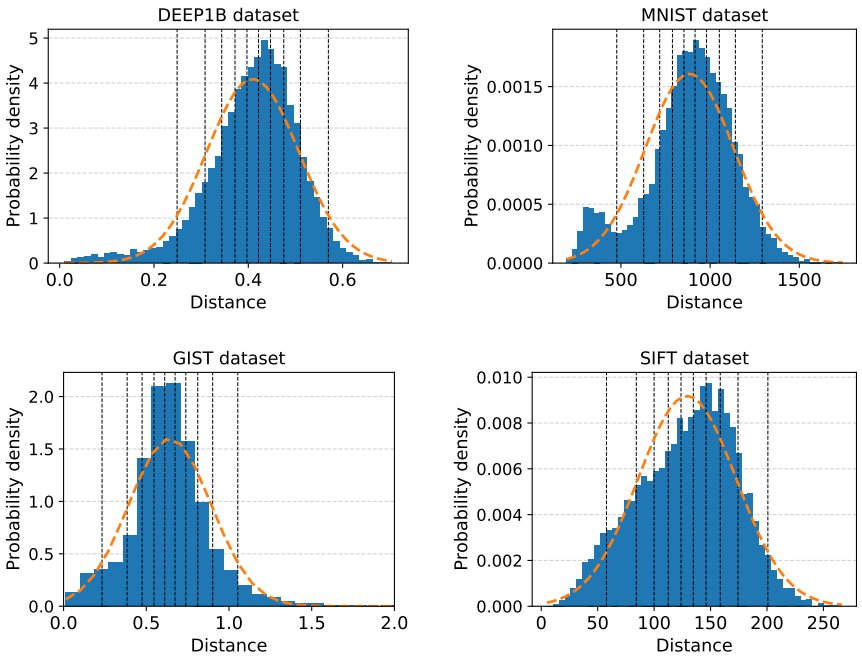

Figure 6-1: Distances to the nearest neighbor over 10,000 training points in each dataset. Orange curve plots the normal approximation that informs the choice of each R_i for the radix bucketing (see Section 3.4.7). The vertical dashed lines represent the choice of each R_i, for $L = 10$ hash tables.

GoogLeNet [109] model. We use the smaller version of DEEP1B containing 10M embeddings. Of these, 9,990,000 are used for training (i.e., building the data structure) and 10,000 are used as test queries [18].

MNIST (Modified National Institute of Standards and Technology dataset) [78] contains feature vector representations of handwritten digits. MNIST is widely used to benchmark ANN search algorithms [10, 12,18,23]. Each feature vector is a 784 dimensional representation of a 28×28 black and white image of a handwritten digit between zero and nine. MNIST contains 60,000 training feature vectors and 10,000 test queries.

Dataset	**Metric**	n	d
DEEP1B	Angular	9,990,000	96
MNIST	Euclidean	60,000	784
GIST	Euclidean	1,000,000	960
SIFT	Euclidean	1,000,000	128

Table 6.1: Summary of dataset characteristics used in our evaluation.

SIFT and GIST [82,94] are both datasets of image descriptors. Both datasets contains

1,000,000 training feature vectors and 10,000 test queries. In SIFT, each feature vector is 128-dimensional image embedding. In GIST, each feature vector is a 960-dimensional embedding. Both datasets are widely used in ANN algorithm benchmarks [12,18,110].

6.2.1 Determining LSH Radii

We determine the radii to use when instantiating our data structure of Figure 3-2 as follows. We sample $10,000$ points from each training set, and find the nearest neighbor among the other training set points. The results are shown in Figure 6-1. We use a normal distribution computed over the resulting distances as an approximation (orange line in Figure 6-1), and choose the R_i along the quantiles of the normal distribution. Figure 6-1 is shown for $L = 10$. This ensures that the number of expected candidates colliding at each R_i is approximately the same (in Figure 6-1, the area between the dashed lines is the same). Our intuition is that equal areas means that each table will have $\frac{1}{L}$ of the collisions, balancing the load and maximizing efficiency. Each vertical dotted line in Figure 6-1 corresponds to the R_i for an LSH family for the ith hash table, where $R_1 > D_{\mathsf{min}}$ and $R_{10} = R_{\mathsf{max}}$.

6.3 Performance Evaluation

In this section we report our accuracy and runtime performance. We compare our results to the state-of-the-art approach for privacy-preserving ANN search.

6.3.1 Protocol Accuracy and Latency

Accuracy. We report the accuracy of Protocol 1 when evaluated on real-world datasets in Figure 6-2. For each dataset, we use the Leech lattice LSH [8] as described in Section 3.4.5. We use $k = 2$ for the amplification factor (see Proposition 1). Accuracy is defined in terms of *recall*: what fraction of approximate nearest neighbors found are at most c-times the distance to the true nearest neighbor [6,57,65].

Matching the theory, increasing the number of tables *or* multi-probes increases

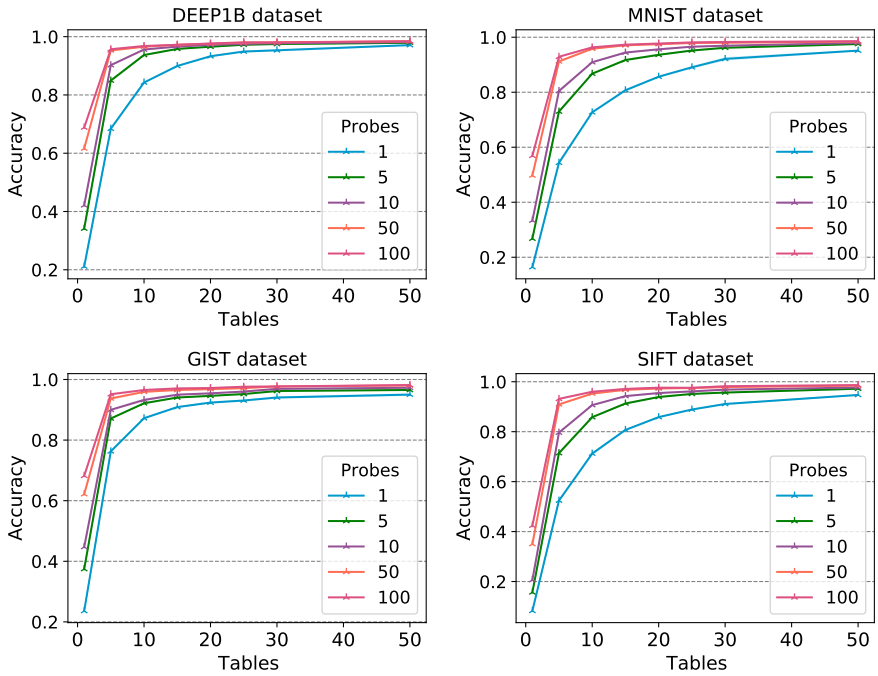

Figure 6-2: Recall (fraction of approximate nearest neighbors found) for different numbers of table multi-probes using the data structure of Figure 3-2 and $c = 2$. Average of 10 trial runs for different numbers of probes. Probes = 1 corresponds to no multi-probing (only retrieving the hash of the query from the table; Section 4.3.1). Radii are distributed as the quantiles of a normal distribution between D_{min} and R_{max} (see Sections 3.4.3 and 6.2.1 for additional details). 95% confidence interval is of size 0.017 and invisible in the plot.

the accuracy. For all datasets we can achieve high accuracy ($> 95\%$) without needing more than 10 tables and 50 multi-probes per table.

Parallelism. The server overhead of answering queries (which involves a linear scan over all hash table keys; see Section 4.4) is easily parallelizable across cores or even across different machines composing each logical server. In our runtime experiments (Figure 6-3), we provide results for both single core and parallelized executions (where each hash table is processed on a separate core). In our experiments, we observe a linear speedup in the degree of parallelism.

Performance. We measure and report the server-side processing time, communication overhead, and total end-to-end latency to answer each ANN query on a dataset, as measured on the client. The server processing time, per hash table, ranges from 108 ms on the MNIST dataset (60,000 items) to approximately 14 s on the DEEP1B dataset

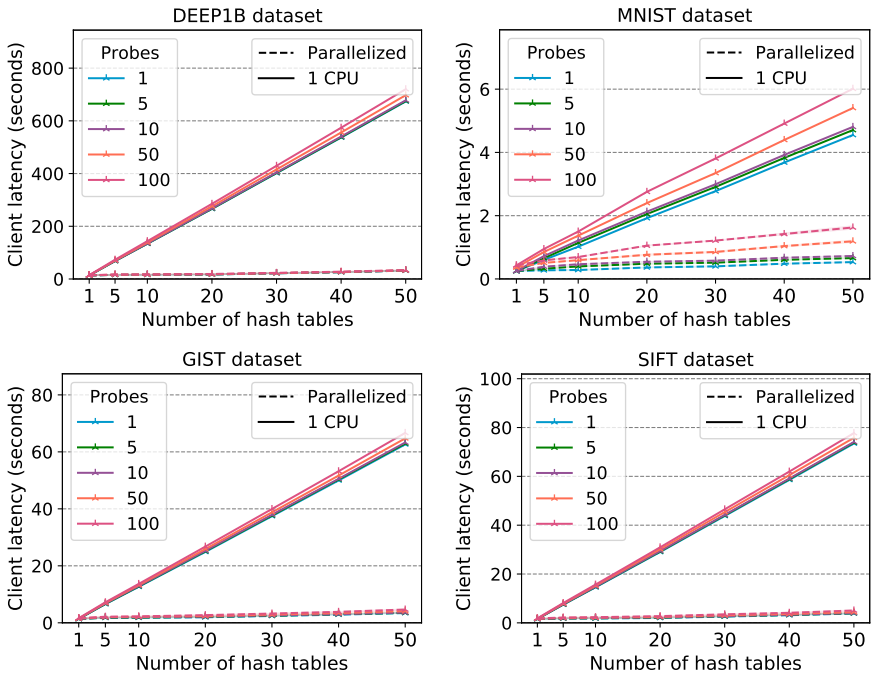

Figure 6-3: Query latency (including server processing time and network delay) as a function of the number of tables and number of multi-probes performed per table. Probes do not increase computation time but do increase communication resulting in increased latency (see our PBR scheme; Section 4.3.2). Server parallelization factor is set to equal the number of hash tables. 95% confidence interval is approximately 500 ms and invisible in the plot.

(10,000,000 items). The processing time is dominated by the DPF. The other steps, including ObliviousMasking, take less than 2 ms. The client processing overhead across all datasets was minimal: never exceeding 10 µs. We report the end-to-end latency (as measured on the client machine) for each dataset in Figure 6-3. We note that the latency includes both the processing time of each server *and* the network delay between the client (located in the `us-west-2` region) and the servers (located in the `us-east-2` region).

Communication. We provide the total communication required per query in Table 6.2. The communication overhead is determined by three factors: (1) the size of the DPF keys, (2) the number of hash tables L, and (3) the number of multi-probes performed per table (recall Section 4.3.2). The size of each DPF key is fixed, given that each key encodes a point function in the range $[0, 2^{64}]$ of table bucket hashes (see Section 4.1). As such, only the number of tables L and the number of probes influence

the total communication in practice. We report the communication overhead for one table as a function of the number of multi-probes in Table 6.2.

Multi-probes:	**1**	**5**	**10**	**50**	**100**
Communication:	4 kB	13 kB	26 kB	123 kB	245 kB

Table 6.2: Communication between the client and both servers per hash table in terms of the number of multi-probes performed.

6.3.2 Comparison to SANNS

We compare our approach to SANNS [38]. We note that Chen et al. [38] only evaluate their approach on the DEEP1B and SIFT datasets, and also use the smaller 10M feature vector version of DEEP1B in their evaluation. We use these datasets for a direct comparison in Section 6.3.2. To the best of our knowledge, SANNS is the most efficient privacy-preserving protocol for ANN search, achieving performance on the order of several seconds when evaluated on both DEEP1B and SIFT (over high-bandwidth network connections). To match the evaluation of SANNS, we compare with two network settings. We note that the network configuration used by SANNS has throughput that is *faster* than what we were able to measure on `localhost` using `iperf3` [62], which capped at 3.6 GB/s. The first setting has network throughput between 40 MB/s to 2.2 GB/s, which we call the "fast" network. The second setting has network throughput between 500 MB/s to 7 GB/s, which we call the "`localhost`" network.[1] Given these network configurations, SANNS is by no means deployable over realistic network connections [108], which are over 30× slower. Because SANNS does not provide an open-source implementation, we use the query times reported in their evaluation and note that our deployment environment resembles theirs (comparable CPUs, network, and degree of parallelism applied). We report the results of our comparison in Table 6.3. Our improvements over SANNS are primarily in terms of communication costs. However, we incur a modest latency overhead on high bandwidth networks, a setting that is highly favorable to SANNS. Our latency is

[1]Chen et al. [38] refer to this as the "fast" network in their evaluation.

Dataset		Protocol 1	SANNS (localhost)	SANNS (Fast Network)
SIFT	Latency (1 CPU):	70.14 s	8.06 s (8.6× ↓)	59.7 s (1.2× ↓)
	Latency (32 CPUs):	2.3 s	1.55 s (1.5× ↓)	14.2 s (6.17× ↑)
	Communication:	3.66 MB	1.77 GB (484× ↑)	
DEEP1B	Latency (1 CPU):	624.3 s	30.1 s (21× ↓)	181 s (3.5× ↓)
	Latency (32 CPUs):	21.33 s	4.58 s (4.6× ↓)	37.2 s (1.75× ↑)
	Communication:	3.66 MB	5.53 GB (1511× ↑)	

Table 6.3: End-to-end comparison between Protocol 1 and SANNS over a 500 MB/s to 7 GB/s network (localhost) and a fast network (40 MB/s to 2.2 GB/s). We fix $L = 30$ hash tables and 50 multi-probes per table. SANNS is network-dominated and hence parallelizes less favorably compared to Protocol 1 (see [38, Table 2]).

between 21× slower to 6× faster in comparison to SANNS. On the other hand, our communication cost is 484–1511× less in comparison. As a result, over slower networks, e.g., average mobile network supporting 12 Mbps [108], SANNS would incur latency ranging between 19 minutes (for SIFT) to an hour (for DEEP1B), just from the network delay. Over such networks, the latency of Protocol 1 is expected to be 160–500× faster in comparison.

Additionally, because bandwidth costs can be upwards of $0.02 per GB [20], while CPU cost is around $0.2 *per hour*, our protocol is monetarily cheaper (per query). SANNS costs $0.04–$0.11 per query just for bandwidth alone. In contrast, our protocol costs up to $0.0002 per query (a 200–550× reduction in total cost).

Chapter 7

Discussion

Using two servers to instantiate lightweight privacy-preserving systems has proven fruitful for a wide range of applications [5,28,39,44,45,47,48,50,75,90], including systems deployed in industry [47,61]. In this thesis, we presented the first lightweight privacy-preserving system for approximate nearest neighbor search using the two-server model. The resulting protocol is highly efficient, with only one round and few megabytes of communication between the client and servers. Moreover, we do not require servers to communicate when answering queries. This is in contrast to many previous uses of the two-server model, which typically also require communication between servers when processing requests. Our evaluation (Chapter 6) provides evidence for the real-world practicality of our approach, especially if deployed on high-latency, low-bandwidth networks and lightweight clients (e.g., low-end smartphones). We survey some potential applications of our protocol.

7.1 Applications and Limitations

Use case 1: Private (approximate) DNA matching. DNA sequencing has gained significant popularity in the last decade, leading to swaths of genome data with a wide array of applications (see Naveed et al. [89] for a survey applications). Such data, while highly valuable, is also highly personal and can uniquely identify individuals [70,89]. One application of DNA sequence matching, highlighted by Naveed

et al. [89, Section 7.1] is personalized healthcare. For example, a patient may be interested in finding whether or not they are prone to a certain disease. Patients submit their genome sequence (query) to an organization (e.g., a pharmaceutical company) mapping sequences to various disease markers (IDs). When queried, the organization finds common disease markers associated with the supplied genome sequence. In this application, both the patient and the organization require privacy. The patient would like to keep their genome sequence private while the organization requires privacy for their potentially proprietary sequences associated with various disease markers [27,53, 112]. The pattern matching can be viewed as a nearest neighbor problem. Indeed, Buhler [33] shows how to apply locality-sensitive hashing to approximate DNA sequence matching. As such, our protocol may be suitable to this application, enabling privacy-preserving services for personalized medicine.

Use case 2: Private targeted advertising. Online targeted advertising has been widely criticized for the rampant privacy violations it incurs [60,92,107]. Companies have been found to use protected demographics like gender, race, or religion to target advertisements in a discriminatory manner [7,35,66,115]. Indeed, recent regulations, introduced in Europe and California, forbid the collection and use of user data for the purpose of advertising without explicit consent [3,4]. Several privacy-preserving systems for targeted advertising have been proposed to address these issues [22,63,72, 96,104,111,114]. The core component of these systems is a privacy-preserving *targeting* mechanism which matches users with relevant ads. This targeting process can be based on LSH [104,114]. For example, in AdVeil [104], single-server PIR is used to query a targeting data structure. While user privacy is maintained, AdVeil does not take database privacy into consideration (which may contain proprietary targeting data). Similarly, with FLoC [114], LSH is used to group users into "cohorts" for which relevant ads are served. The smaller the cohort, the less privacy users can expect from the system, resulting in an undesirable privacy-accuracy trade-off in the ad targeting mechanism. Both these systems could potentially be extended with our approach to similarity search for the purpose of introducing better database and user privacy.

Use case 3: Private medical diagnoses. Zuber and Sirdey [121] highlight a use case involving classifying images for medical diagnoses. They show how their nearest neighbor classifier can be used to (privately) classify tumors as either malignant or benign using the Breast Cancer Wisconsin diagnostic dataset [17]. Zuber and Sirdey [121] perform this classification using fully-homomorphic encryption, (see Section 2.3). Our protocol could be applied to this setting, with very minimal overhead. For example, the Breast Cancer dataset contains 569 feature vectors. We can assign the ID of each feature vector to be either 0 or 1, for malignant or benign. Finding the nearest neighbor in this dataset can be performed using Protocol 1 in under 10 ms of end-to-end latency. Even on much larger datasets (e.g., MNIST with 60,000 feature vectors) latency is under 900ms per query, as was shown in Chapter 6.

Limitations. The primary limitation of our approach is that it incurs greater database leakage compared to the ideal functionality. We show that the database leakage is asymptotically optimal, but concretely a small factor worse on real data; see empirical analysis in Chapter 5). While we believe that this leakage is necessary for efficiency, since it eliminates the need for oblivious comparisons, it is an inherent limitation associated with our protocol. In settings where this leakage is deemed too high, it may become necessary to combine our approach with secure function evaluation techniques for instantiating oblivious comparisons. While this would impact performance, we observe that the number of comparisons needed to eliminate leakage is at most $\sqrt{n}$ (the size of the candidate set). This is in stark contrast to other systems (e.g., SANNS [38]) where a much larger number of comparisons are required (see Section 2.3).

7.2 Future Work

We outline some potential direction for future work.

Better concrete leakage bounds. Deriving better (concrete) leakage bounds under additional distance metrics and locality-sensitive hash families would be a good direction for future work. Specifically, while we arrive at our concrete leakage in Section 5.2.2 by considering the Leech-lattice LSH for Euclidean distance, it could be

interesting to analyze other hash families that may further minimize leakage. While we eschew oblivious comparisons through our radix bucketing approach described in Section 3.4.3, as we identify in Chapter 5, this process inherently results in more leakage. Finding alternative ways to avoid oblivious comparisons can be another direction for reducing leakage. For example, a different approach to reducing leakage would be to combine our protocol with existing two-party computation techniques to instantiate oblivious comparisons. We outline such an approach next.

Reducing leakage through two-party computation. A natural approach to reducing leakage in our protocol would be to instantiate two-party computation between the servers. Doing so would prevent the client from learning which radix bucket the nearest neighbor appears in relative to the query. While this approach would introduce communication (in our protocol the server do not communicate), it is an effective way of reducing leakage. We note, however, that most of the "heavy-lifting" is performed by our protocol, since the communication between servers to extract the nearest neighbor from the candidate set is $O(\sqrt{n})$. In contrast to SANNS, where the communication is $O(n)$ in the worst-case, this idea would result in significant practical gains compared to performing oblivious comparisons over the entire database.

Extensions to other distance and similarity metrics. In this thesis, we focused specifically on the Euclidean distance metric, which encompasses a wide array of applications. It would be interesting to explore the application of our protocol to other distance metrics (e.g., hamming distance) or similarity metrics (e.g., Jaccard similarity). Our approach is general and supports any suitable locality-sensitive hash function. As such, while we do not explicitly evaluate our approach on more distance metrics, we have reason to expect similar results. Additionally, we note that our worst-case leakage bounds (Claim 3) are general and apply to any distance or similarity metric. However, the concrete leakage under other metrics would require further analysis, and could follow similar ideas to our concrete leakage bounds.

Single-server setting. Perhaps the most interesting direction for future work would be to apply the techniques presented in this thesis to a single-server setting. We

observe that all techniques we present (i.e., oblivious masking, partial batch retrieval, etc.) translate naturally to the single-server setting. The only caveat, however, is that we are not aware of any PIR-by-keyword schemes achieving the same properties that we exploit from distributed point functions (Section 4.1). PIR-by-keyword can be instantiated in the single-server setting (see Chor et al. [41]) but requires revealing the keys to the client in the process. This is unacceptable in our protocol, as it would introduce additional leakage: the client learns which hashes are in each of the hash tables. This is where we rely on distributed point functions, which do not necessitate revealing the set of keywords to the client. Finding ways to either (1) query the hash table without leaking to the client which hash keys are present or (2) developing a single-round PIR-by-keyword scheme in the single-server setting which achieves the same properties as described Section 4.1, would immediately make all the ideas presented in this thesis applicable to the single-server setting.

7.3 Conclusions

We presented a new protocol for privacy-preserving similarity search based on locality-sensitive hashing. Our protocol is lightweight, incurring almost no processing overhead on the client while remaining concretely efficient for the database servers. We showed that clients can obtain answers to queries within a few seconds of latency, even on slow networks and large databases. Compared to the state-of-the-art in privacy-preserving approximate neighbor search, which requires many rounds and *gigabytes* of communication, our protocol requires only one round and less than four *megabytes* of communication. We achieve these gains by tolerating some extra database leakage to the client (while maintaining complete client privacy). However, we carefully analyze and bound the database leakage, which we show is asymptotically optimal (matching the ideal functionality) but concretely reveals a bit more information on the database. We show that this leakage is bounded, even in the face of malicious clients deviating from protocol. Finally, we surveyed several applications of our protocol, which range from privacy-preserving personalized medicine to targeted advertising.

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