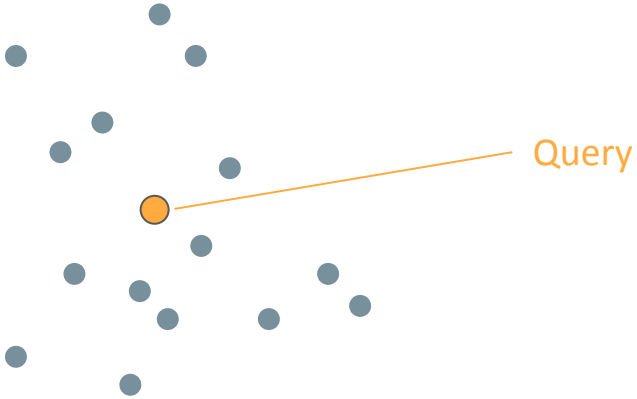
Private Approximate Nearest Neighbor Search with Sublinear Communication

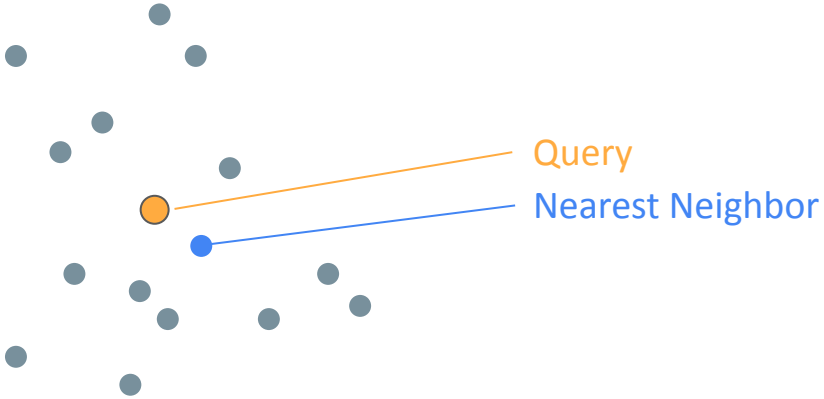
Sacha Servan-Schreiber
with Simon Langowski and Srini Devadas

MIT CSAIL

Nearest Neighbor (NN) search

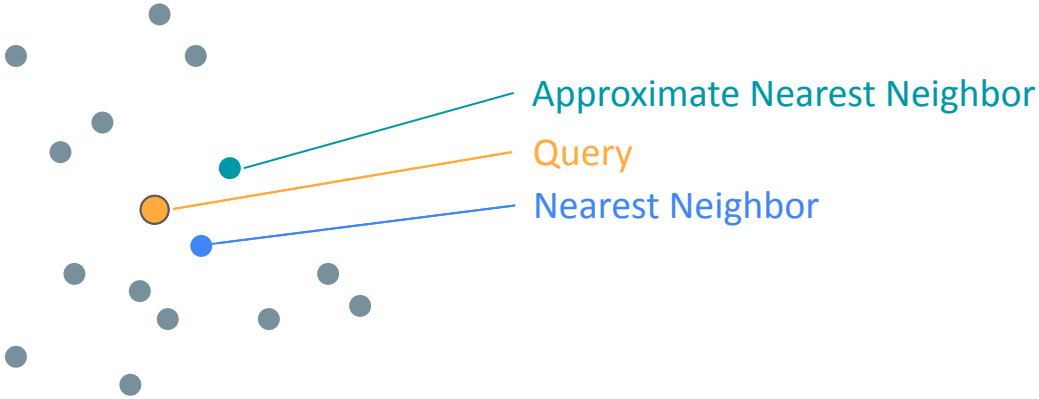


Nearest Neighbor (NN) search



Approximate Nearest Neighbor (ANN) search

(standard relaxation used in practice)



Example: music recommendations [1]

[1]: https://github.com/spotify/annoy

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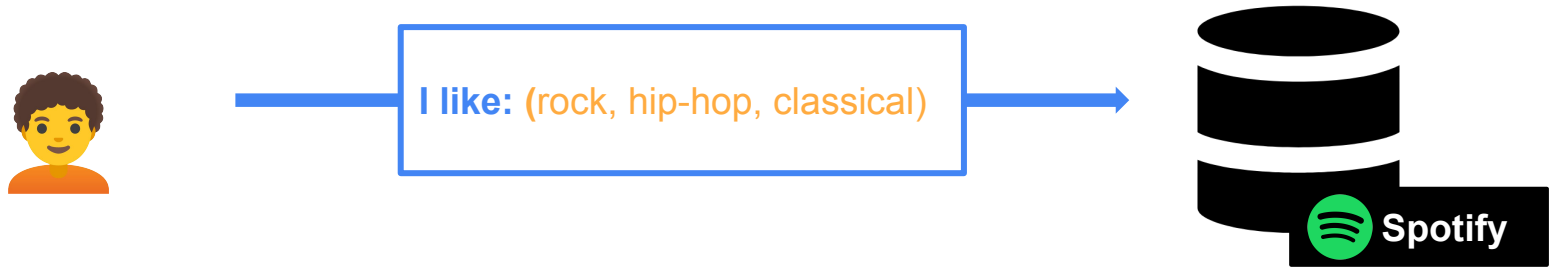

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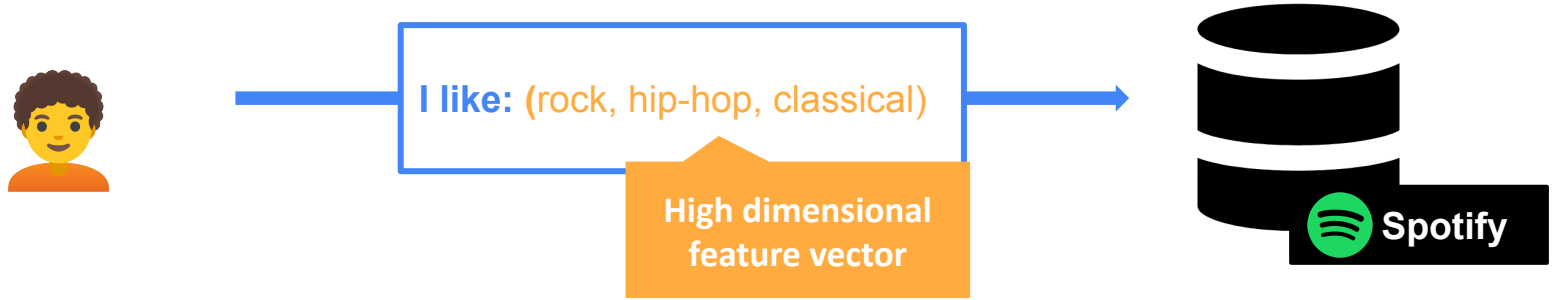

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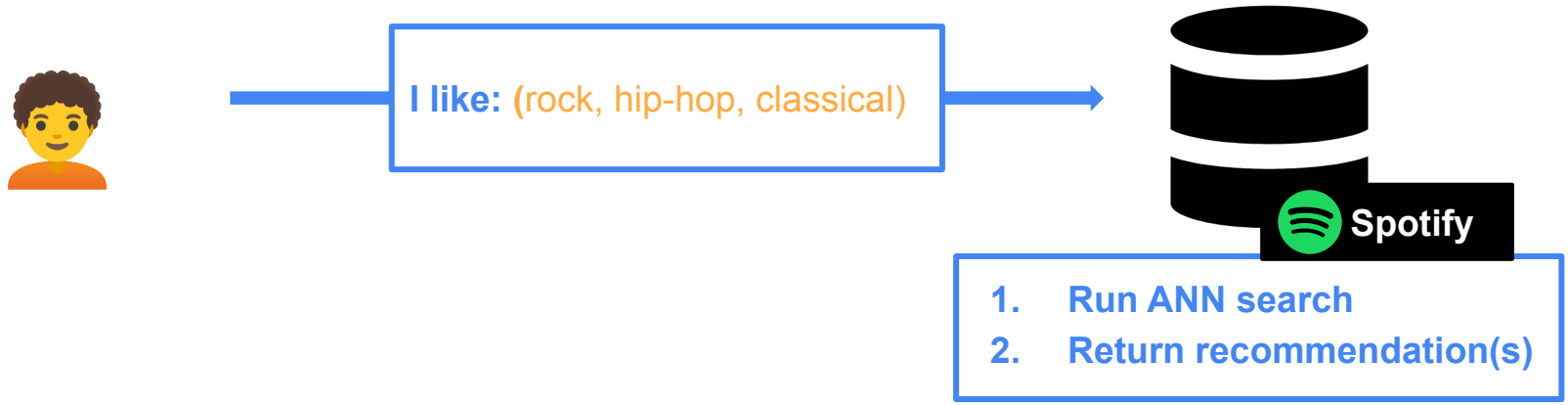

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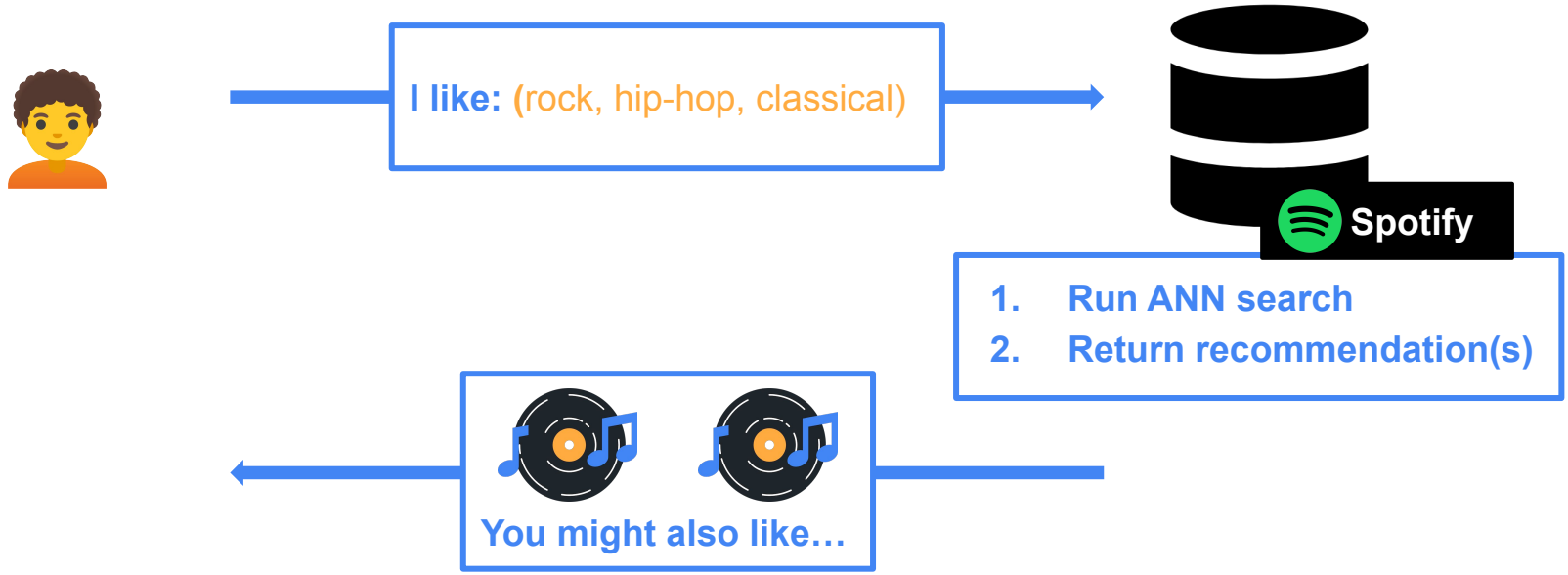

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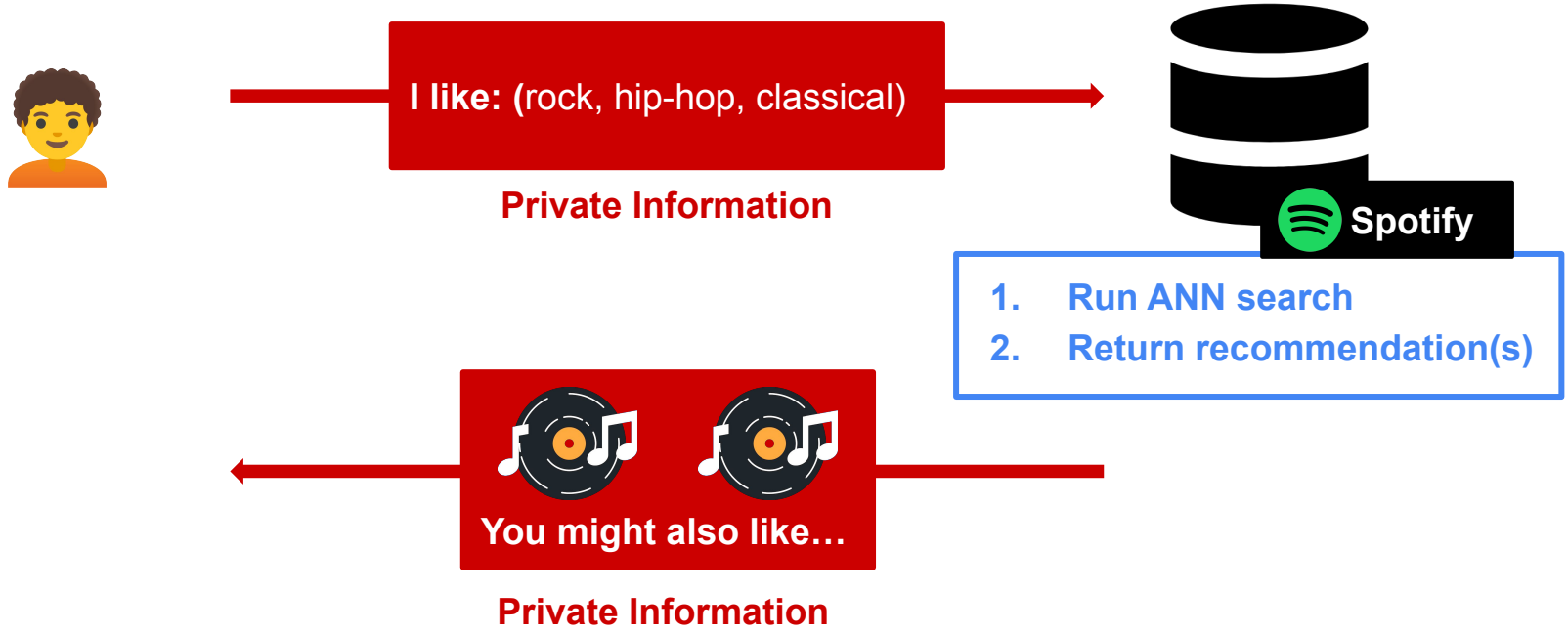

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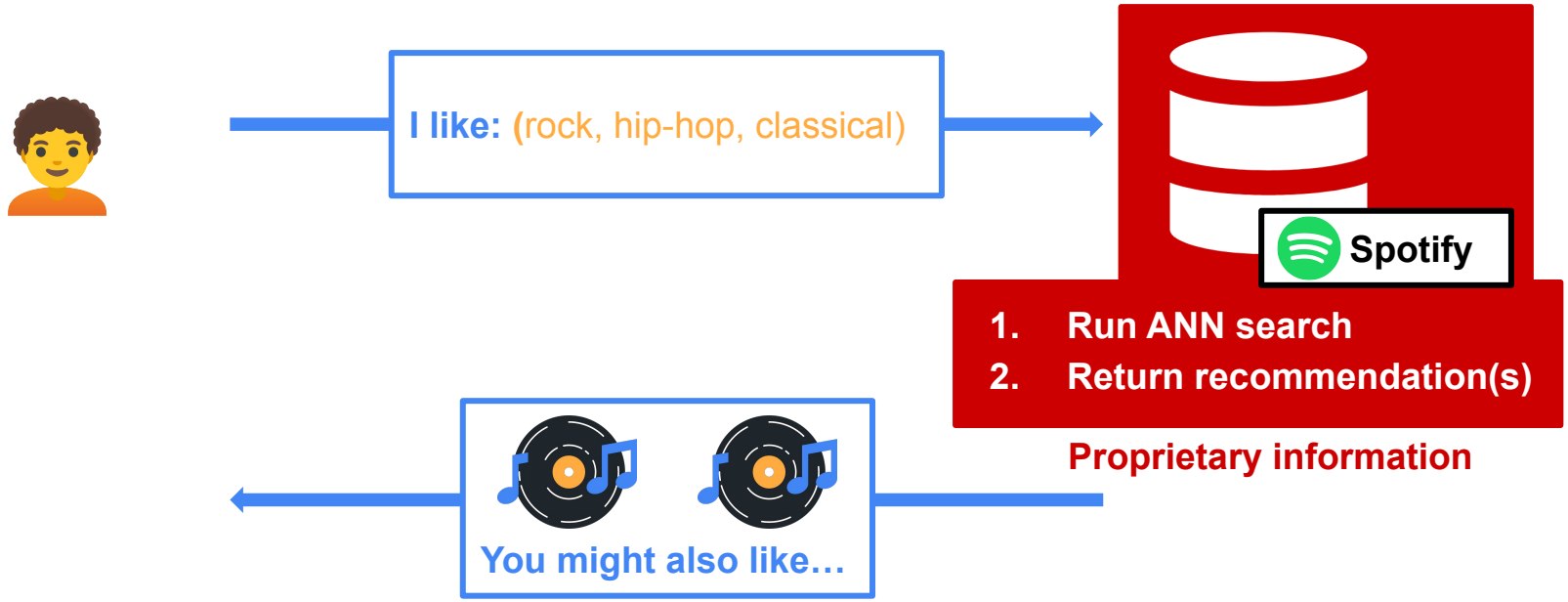

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Prior work on private ANN search

[1]: Chen, Hao, et al. "SANNS: Scaling Up Secure Approximate k-Nearest Neighbors Search." *29th USENIX Security Symposium (USENIX Security 20)*. 2020.
[2]: Shaul, Hayim, Dan Feldman, and Daniela Rus. "Secure-ish Nearest Neighbors Classifier." *Proceedings on Privacy Enhancing Technologies* 2020.3 (2020): 42-61.

Prior work on private ANN search

Two-party Computation

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Two-party Computation

- **Not** lightweight:
  - **1 to 5 GB of communication** with databases of 1,000,000 items [1].

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  - A **malicious** database can deviate from protocol and learn the query.
  - A **malicious** client can deviate from protocol and recover the database.

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Fully-homomorphic Encryption

- **Is** lightweight for the client.

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- **Is** lightweight for the client.
- **Is** *trivially* maliciously-secure.

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Fully-homomorphic Encryption

- **Is** lightweight for the client.
- **Is** *trivially* maliciously-secure.
- **Not** lightweight for the database.
  - Takes **hours** with small databases of e.g., 500 to 2,000 items [2].

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Prior work on private ANN search

Two-party Computation

Fully-homomorphic Encryption

Our goals:

(1) **low communication** for the client,

(2) concrete **efficiency** for the database,

(3) **privacy** for the client and the database,

(4) and **malicious security**.

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Our work

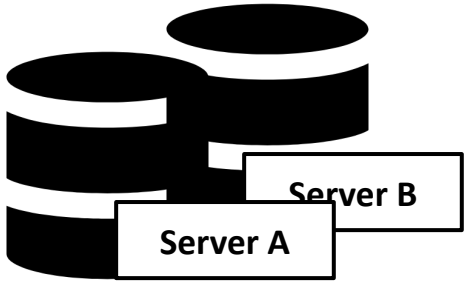
The setting: two non-colluding database servers.

Q: Why do we need non-colluding servers?

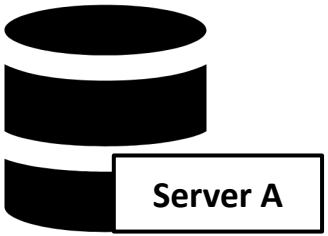
A: For efficient, symmetric-key cryptography only.[1]

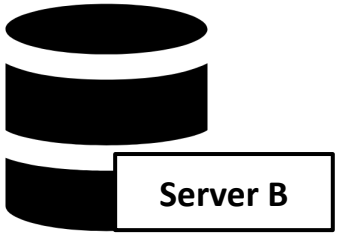
[1] See Appendix E of the full version of our paper [1] for a **single-server protocol** that is less concretely efficient but doesn't require any trust assumptions.

[1]: Full version of our paper: https://eprint.iacr.org/2021/1157.pdf.



Our work





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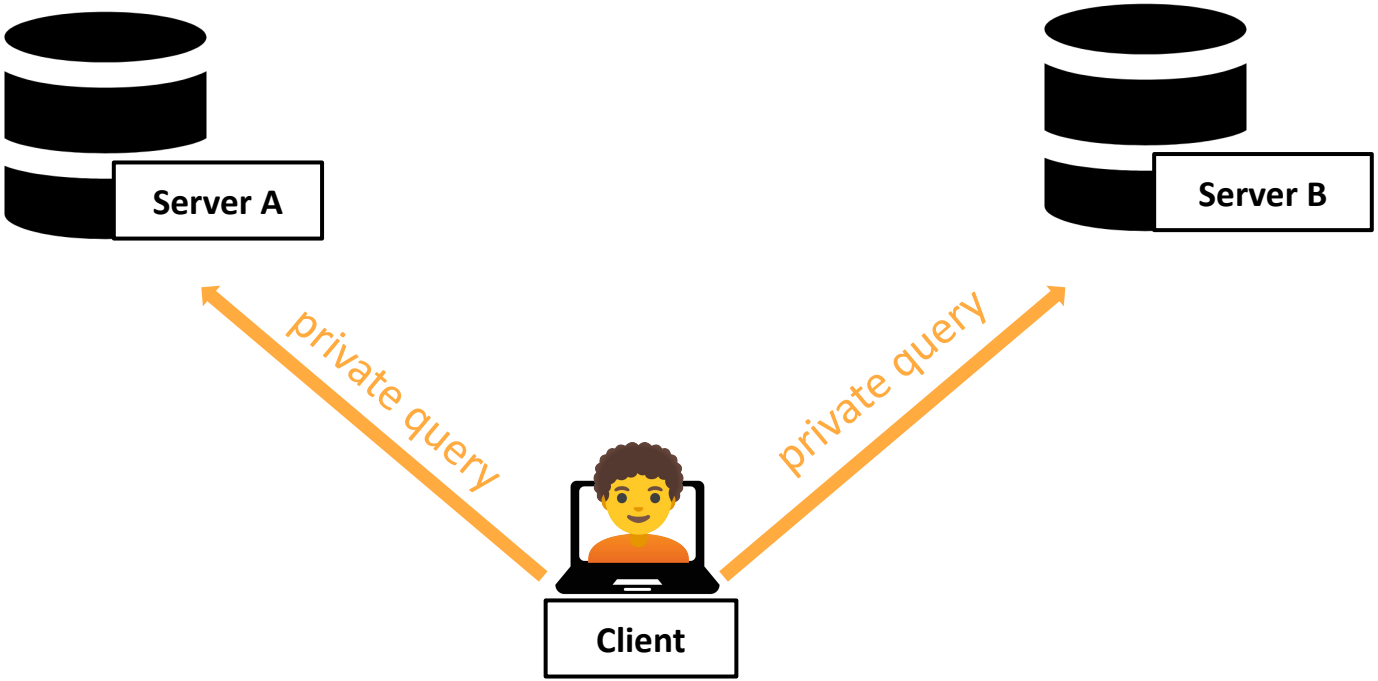

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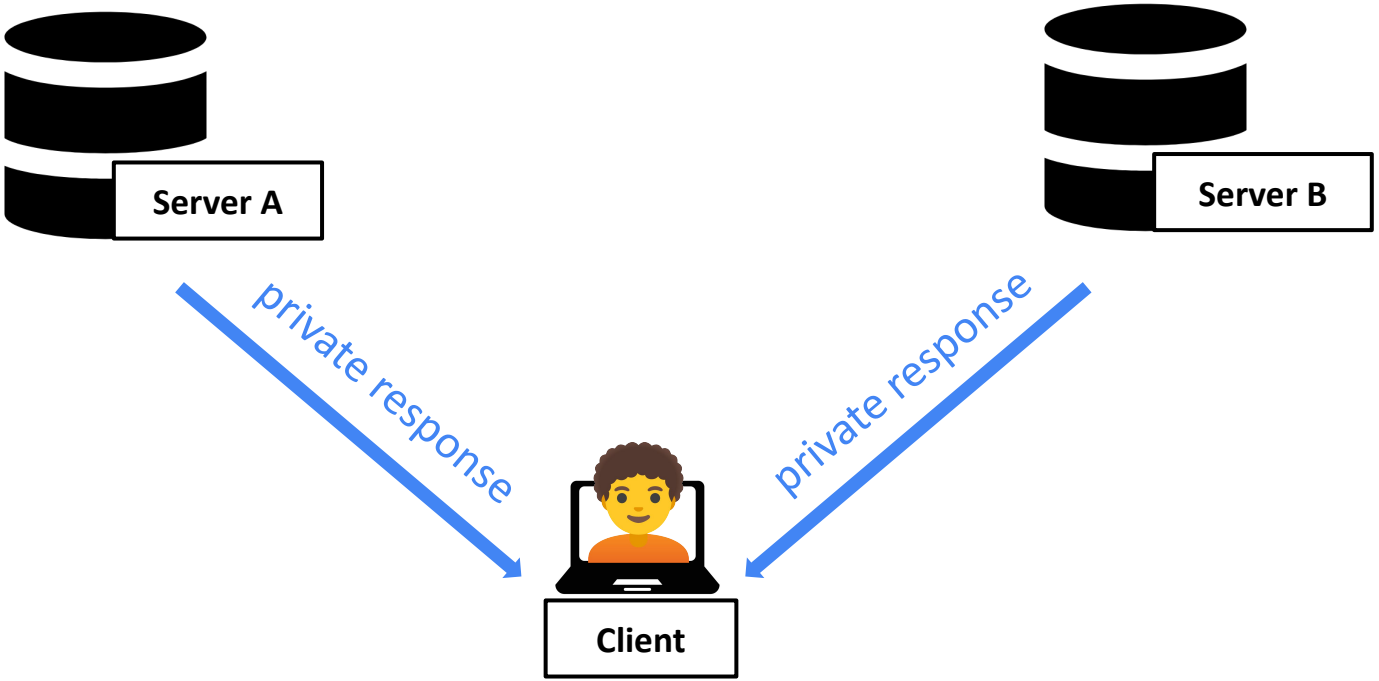
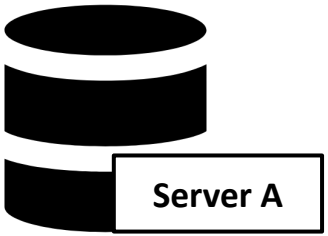
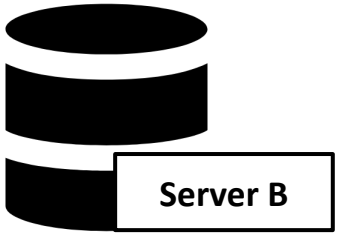
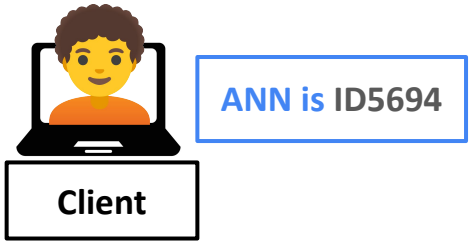

Server A
Server B
private response
private response
Client

Our work







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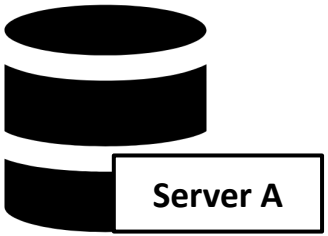
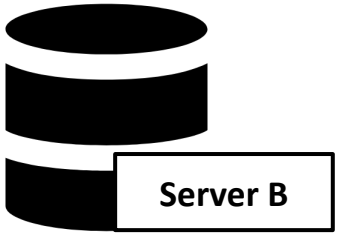
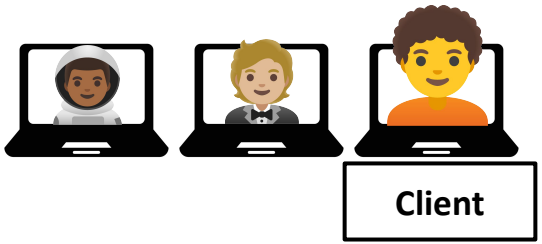
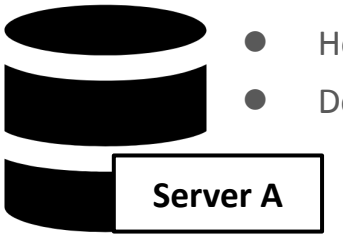




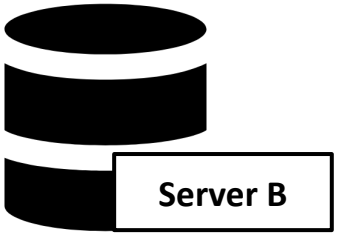


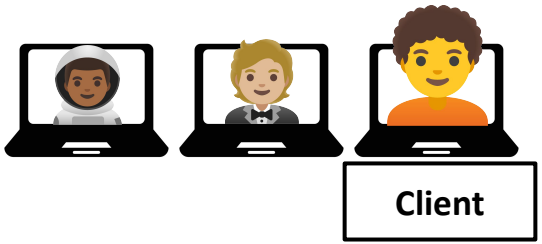
Threat model

Servers



- Hold replicas of the database.
- Do not collude with clients or one another.





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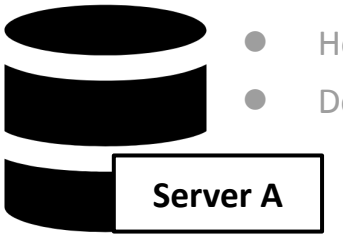

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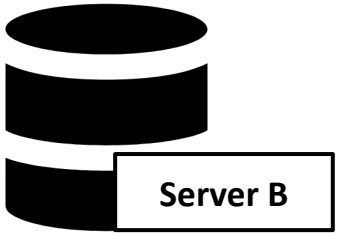
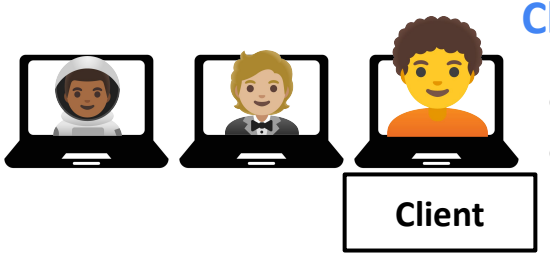


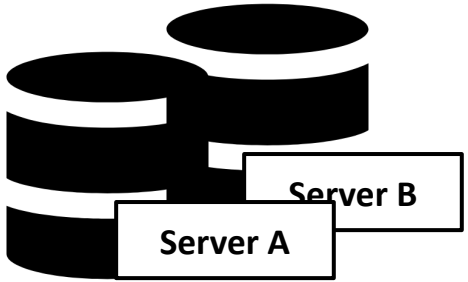
Clients

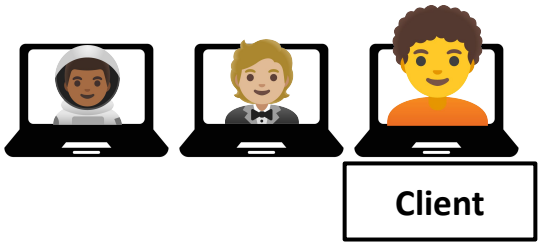


- Will try to learn as much as possible about the database.
- May collude with other malicious clients.

Threat model

Guarantees

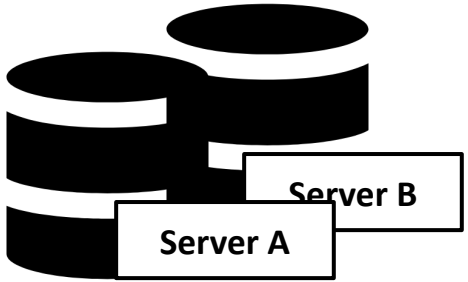


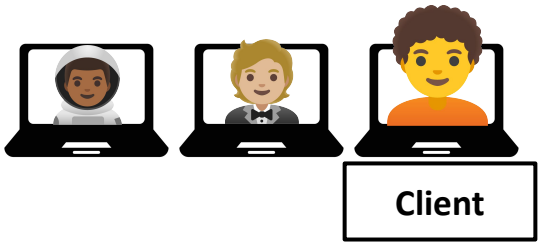


Threat model

Guarantees

- **Accuracy** if both servers follow the protocol.

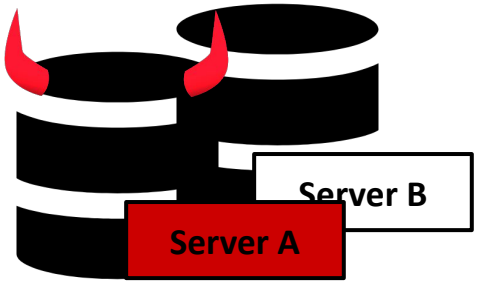


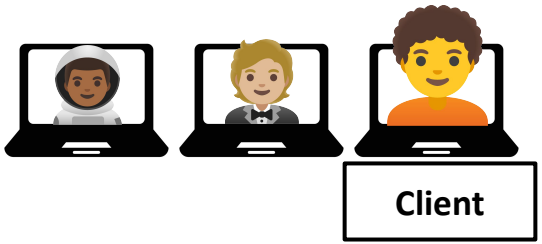


Threat model

Guarantees

- **Accuracy** if both servers follow the protocol.
- **User privacy** even if a server is malicious.

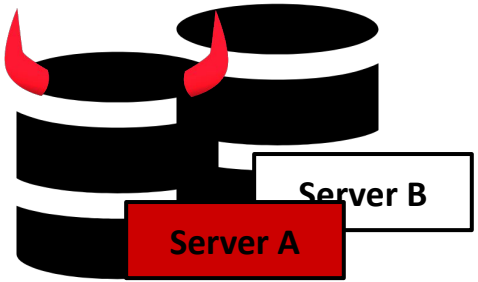


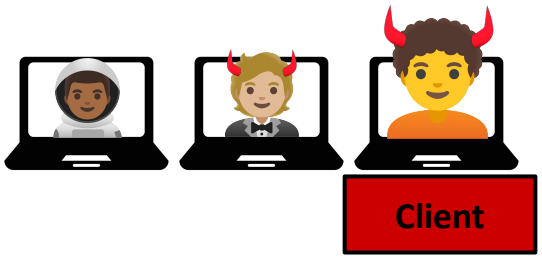


Threat model

Guarantees

- **Accuracy** if both servers follow the protocol.
- **User privacy** even if a server is malicious.
- **Database privacy** even if a subset of clients are malicious.

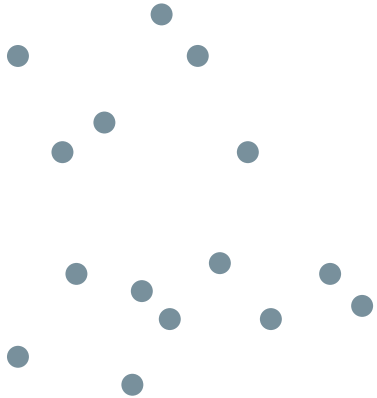




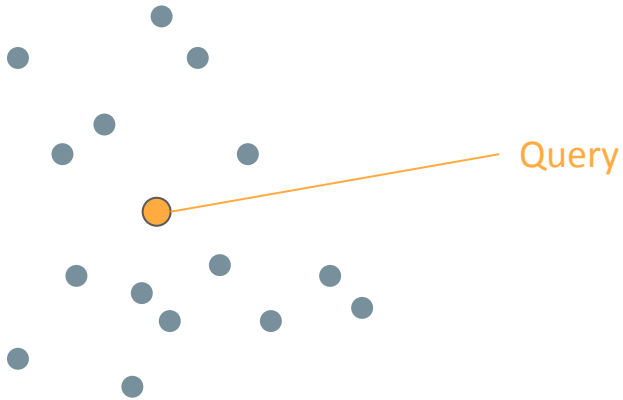
Finding Nearest Neighbors

(non-privately, using Locality-Sensitive Hashing)

Finding the ANN using LSH (non-privately)



Finding the ANN using LSH (non-privately)



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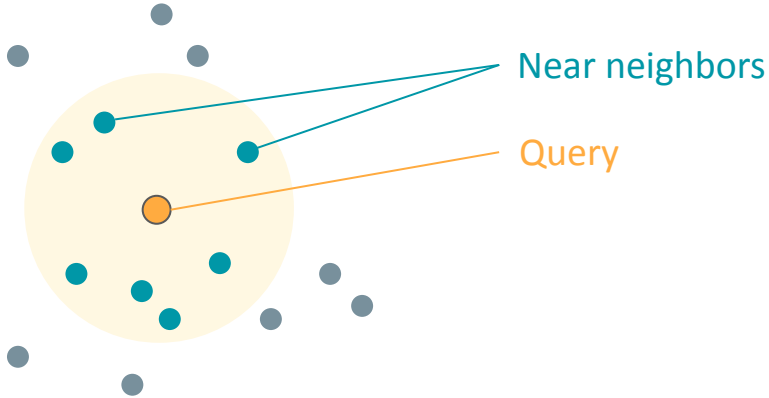

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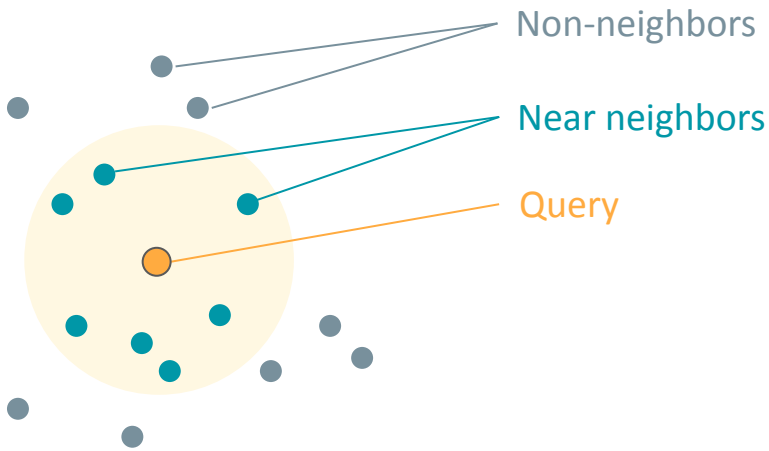

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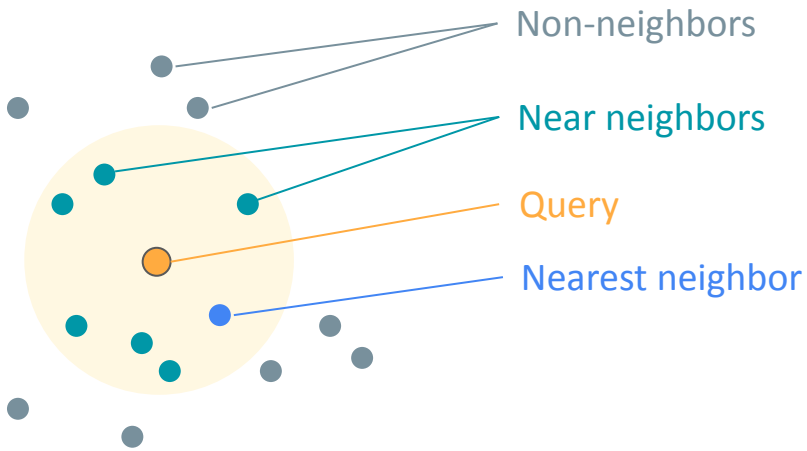

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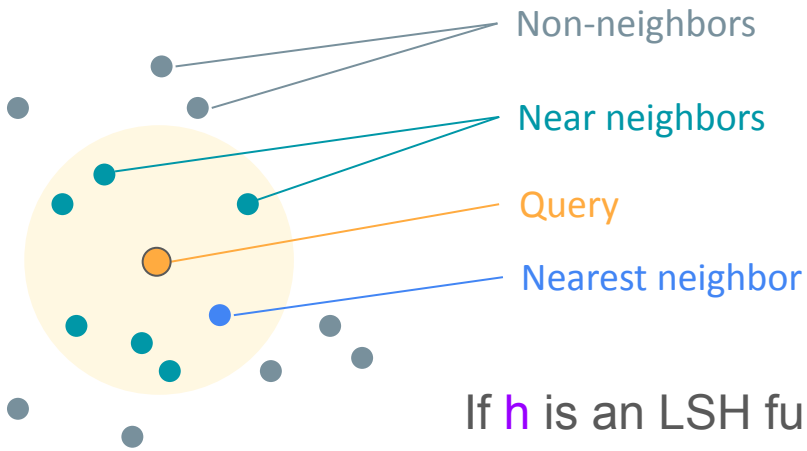
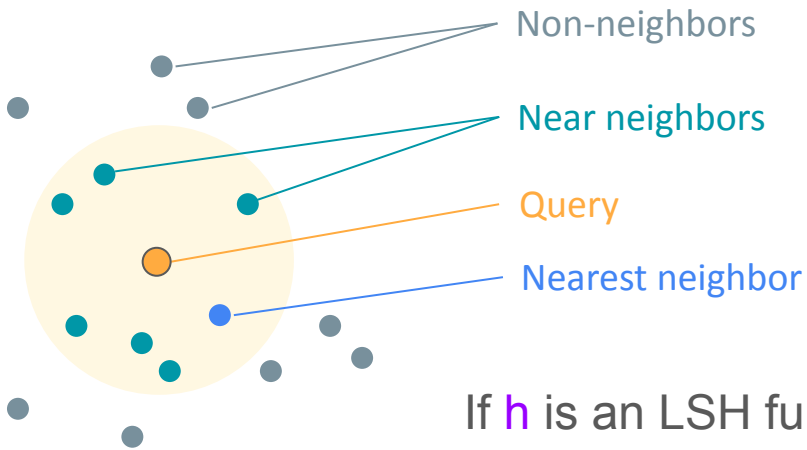


If h is an LSH function:

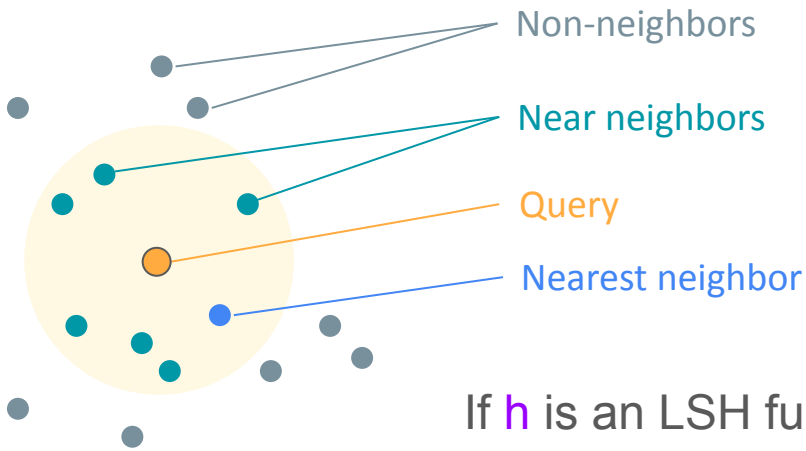
Finding the ANN using LSH (non-privately)



If h is an LSH function:

Pr[h(●) = h(●)] is high

Finding the ANN using LSH (non-privately)



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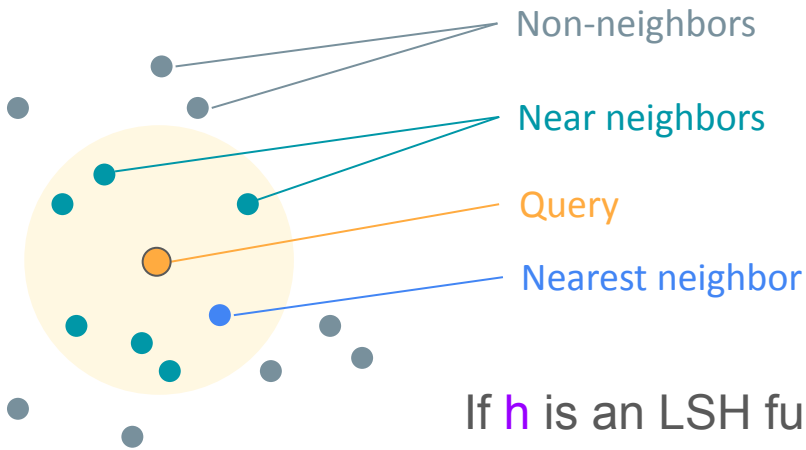

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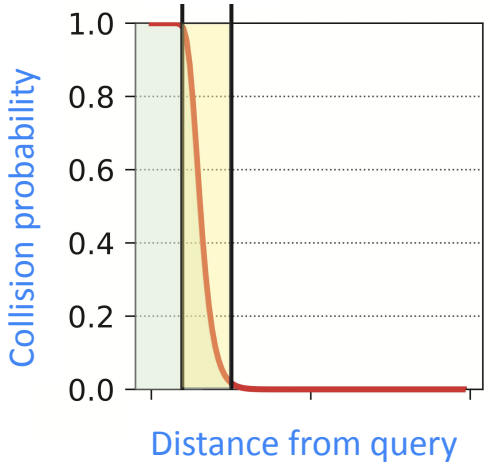
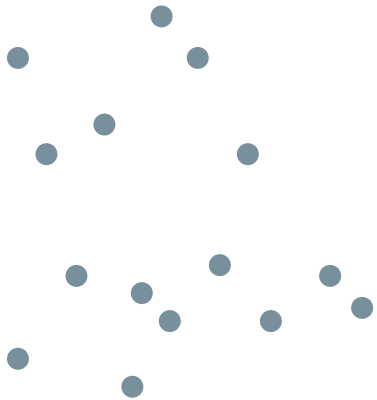
Pr[h(●) = h(●)] is high

Pr[h(●) = h(●)] is low

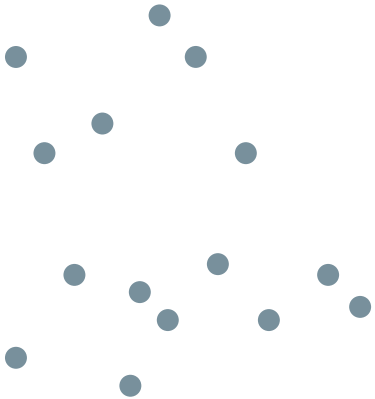


Step 1: Build an LSH hash table using h.



Data

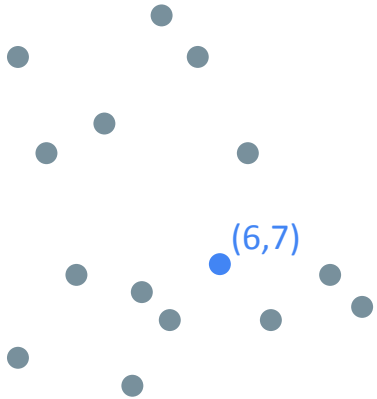
Step 1: Build an LSH hash table using h.



Data

Hashkey	Value(s)

Step 1: Build an LSH hash table using h.

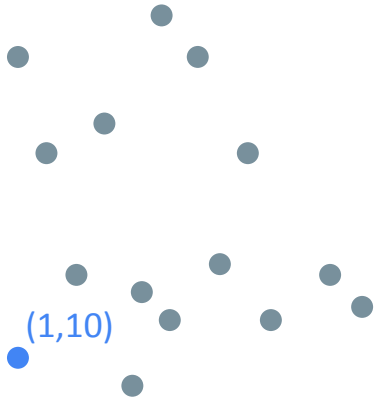


Data

h(●)

Hashkey	Value(s)
09ac34fd	ID3: (6,7)

Step 1: Build an LSH hash table using h.

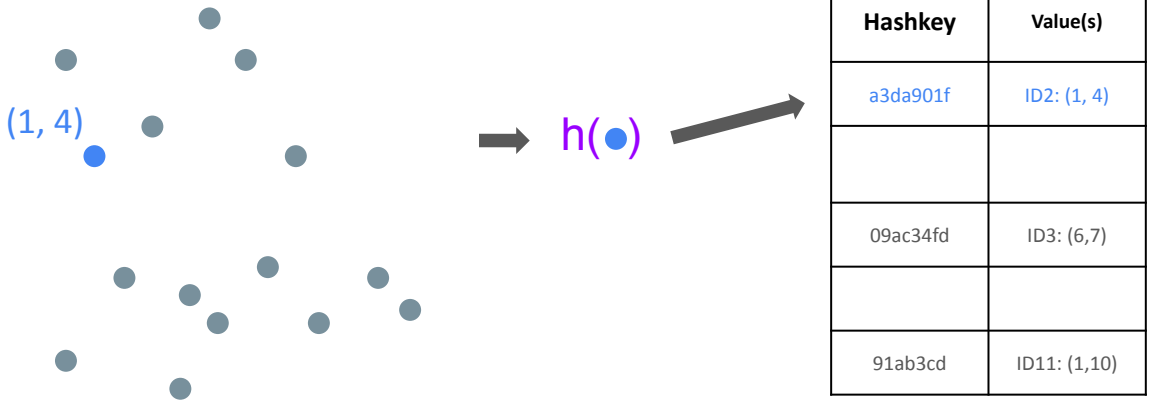


Data

h(●)

Hashkey	Value(s)
09ac34fd	ID3: (6,7)
91ab3cd	ID11: (1,10)

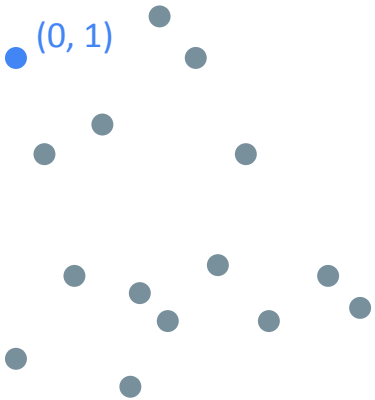
Step 1: Build an LSH hash table using h.



Hashkey	Value(s)
a3da901f	ID2: (1, 4)
09ac34fd	ID3: (6,7)
91ab3cd	ID11: (1,10)

Data

Step 1: Build an LSH hash table using h.



Data

h(●)

Hashkey	Value(s)
a3da901f	ID2: (2, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
91ab3cd	ID11: (1,10)

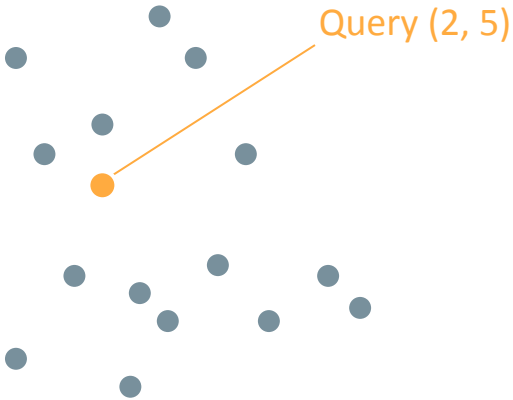
Step 1: Build an LSH hash table using h.

Data

Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
⋮	⋮
91ab3cd	ID11: (1,10)

Step 1: Build an LSH hash table using h.

Step 2: Query the hash table.



Data

Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
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⋮	⋮
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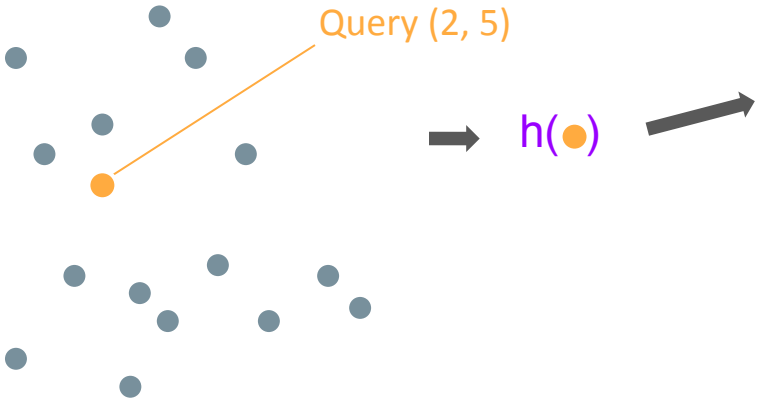

Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
⋮	⋮
91ab3cd	ID11: (1,10)

Data

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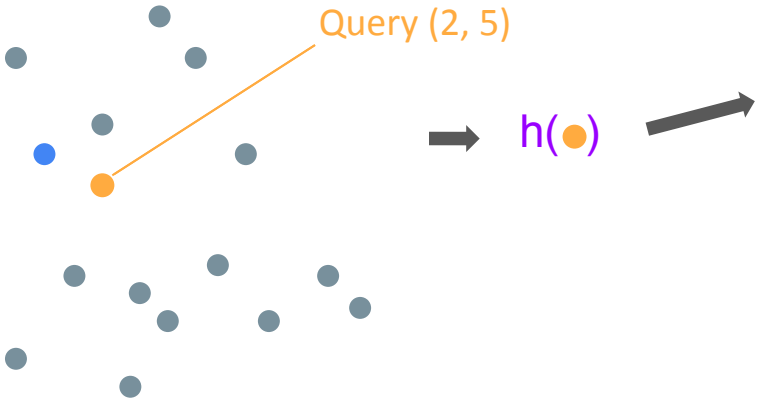


Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
⋮	⋮
91ab3cd	ID11: (1,10)

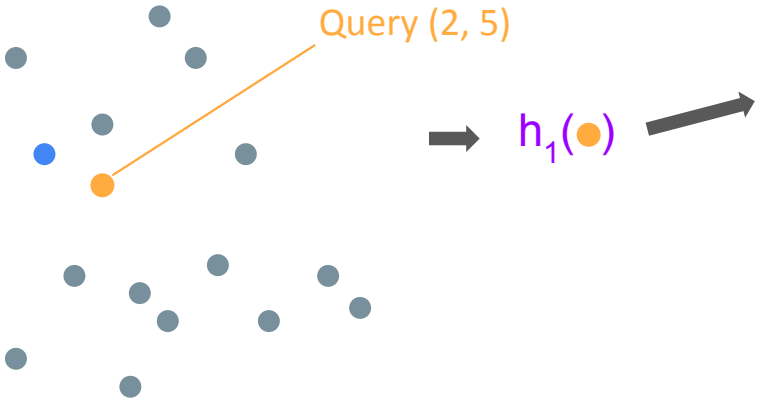
Data

Candidate set: {ID2: (1,4)}

Step 1: Build an LSH hash table using h.

Step 2: Query the hash table.

Step 3: Repeat with many different hash tables.



Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
⋮	⋮
91ab3cd	ID11: (1,10)

Hashkey	Value(s)
beda11fe	ID1000000
f12fbe10	ID101, ID949
ac33445a	ID5694
⋮	⋮
91ab3cd	ID11, ID9191

Hashkey	Value(s)
aa423ef	ID900
10acdff	ID10
ff1e3432	ID514
⋮	⋮
f2f2e12	ID141

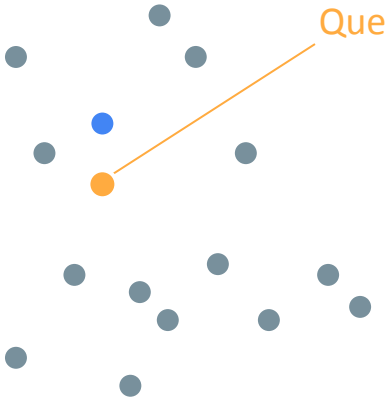
Data

Candidate set: {ID2: (1,4)}

Step 1: Build an LSH hash table using h.

Step 2: Query the hash table.

Step 3: Repeat with many different hash tables.



Data

$h_2(\bullet)$

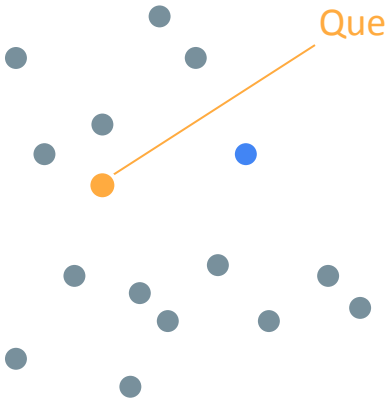
Hashkey	Value(s)
beda11fe	ID10: (9,10)
f12fbe10	ID101: (5,0)
ac33445a	ID5694: (2,4)
⋮	⋮
91ab3cd	ID11: (4,8)

Candidate set: {ID2: (1,4), ID5694: (2,4)}

Step 1: Build an LSH hash table using h.

Step 2: Query the hash table.

Step 3: Repeat with many different hash tables.



Data

$h_3(\bullet)$

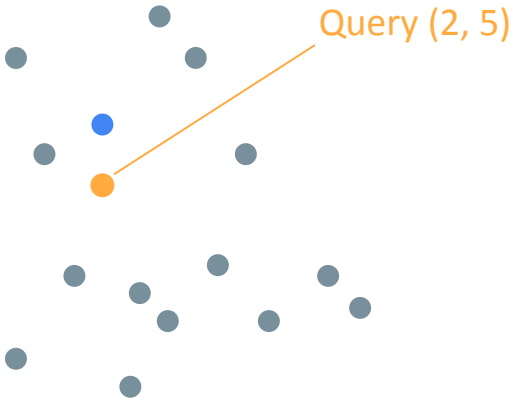
Hashkey	Value(s)
a3da901f	ID2: (1, 4)
c26fab1d	ID100: (0,1)
09ac34fd	ID3: (6,7)
⋮	⋮
91ab3cd	ID11: (1,10)

Hashkey	Value(s)
beda11fe	ID10: (9,10)
f12fbe10	ID101: (5,0)
ac33445a	ID5694: (2,4)
⋮	⋮
91ab3cd	ID11: (4,8)

Hashkey	Value(s)
aa423ef	ID900: (3,4)
10acdff	ID10: (9,10)
ff1e3432	ID514: (8,8)
⋮	⋮
f2f2e12	ID141: (10, 8)

Candidate set: {ID2: (1,4), ID5694: (2,4), ID900: (3,4)}

Step 1: Build an LSH hash table using h.
Step 2: Query the hash table.
Step 3: Repeat with many different hash tables.
Step 4: Find closest neighbor in the candidate set.



Data

Hashkey	Value(s)
a3da901f	ID2: (1, 4)
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⋮	⋮
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Hashkey	Value(s)
beda11fe	ID10: (9,10)
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Hashkey	Value(s)
aa423ef	ID900: (3,4)
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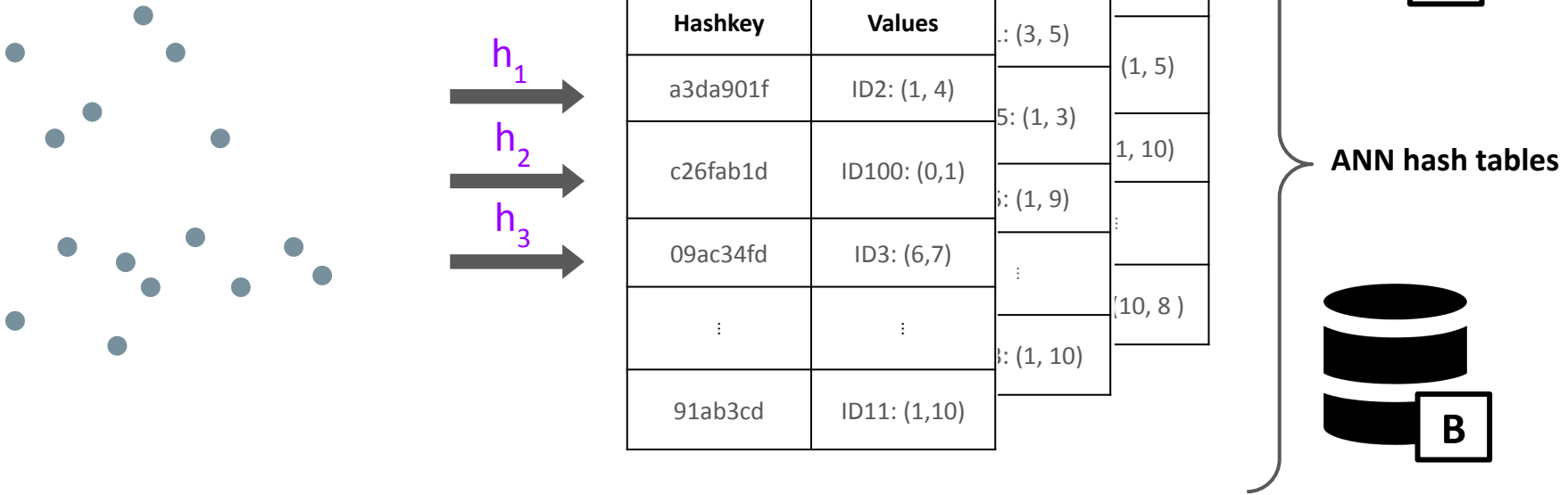
f2f2e12	ID141: (10, 8)

Candidate set: {ID2: (1,4), **ID5694**: (2,4), ID900: (3,4)}

A strawman protocol

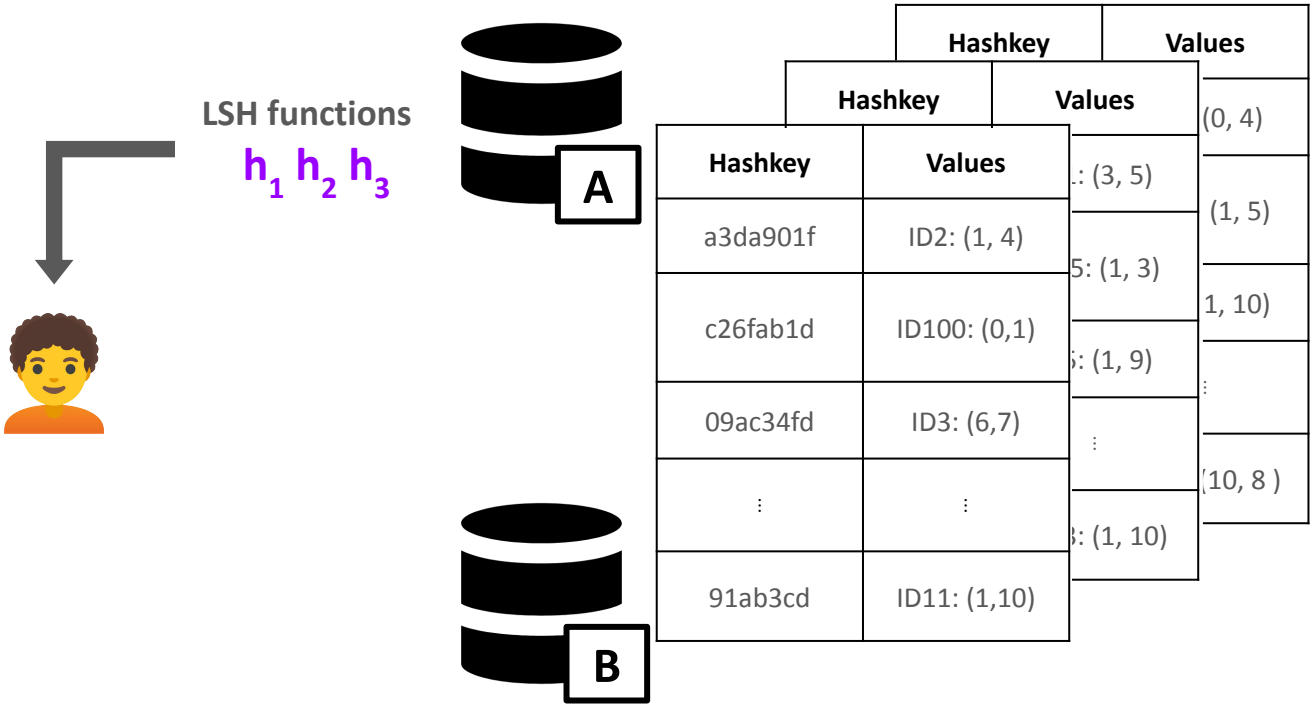
A strawman protocol

One-time setup: Construct LSH tables



A strawman protocol

Step 1: Client uses LSH functions to find the hashkey



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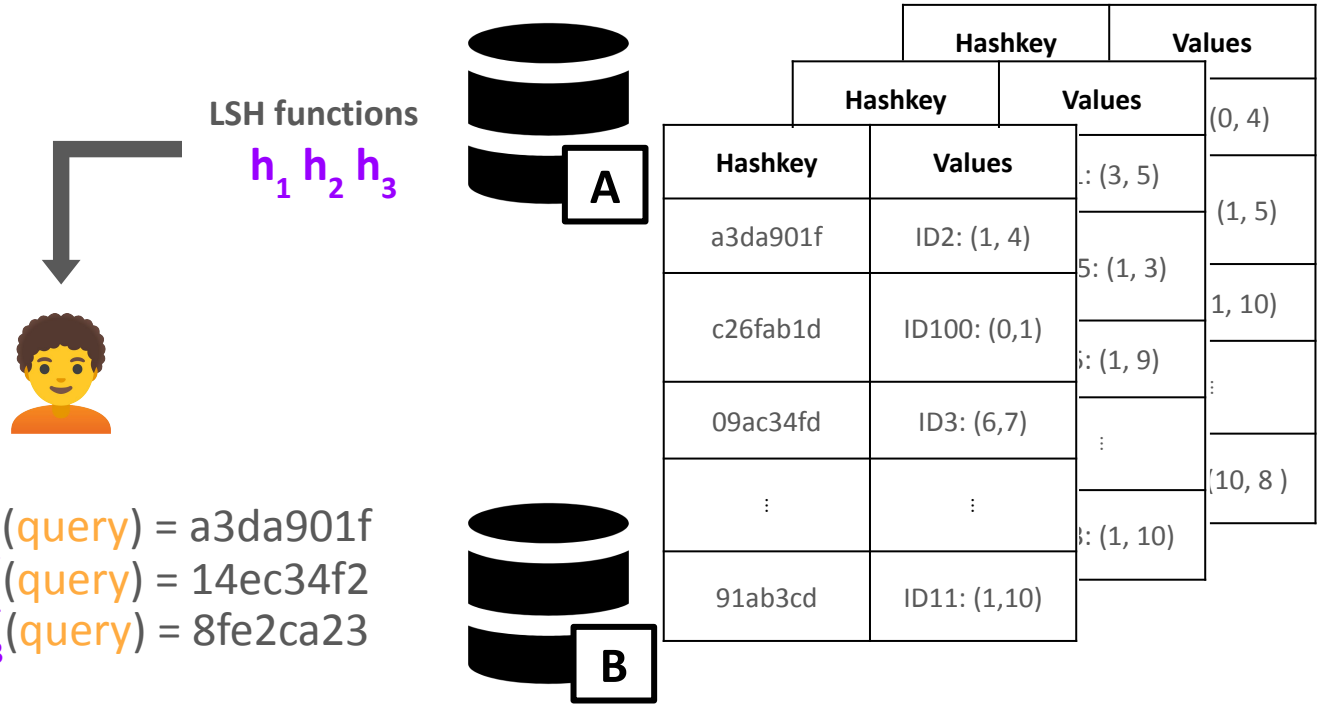

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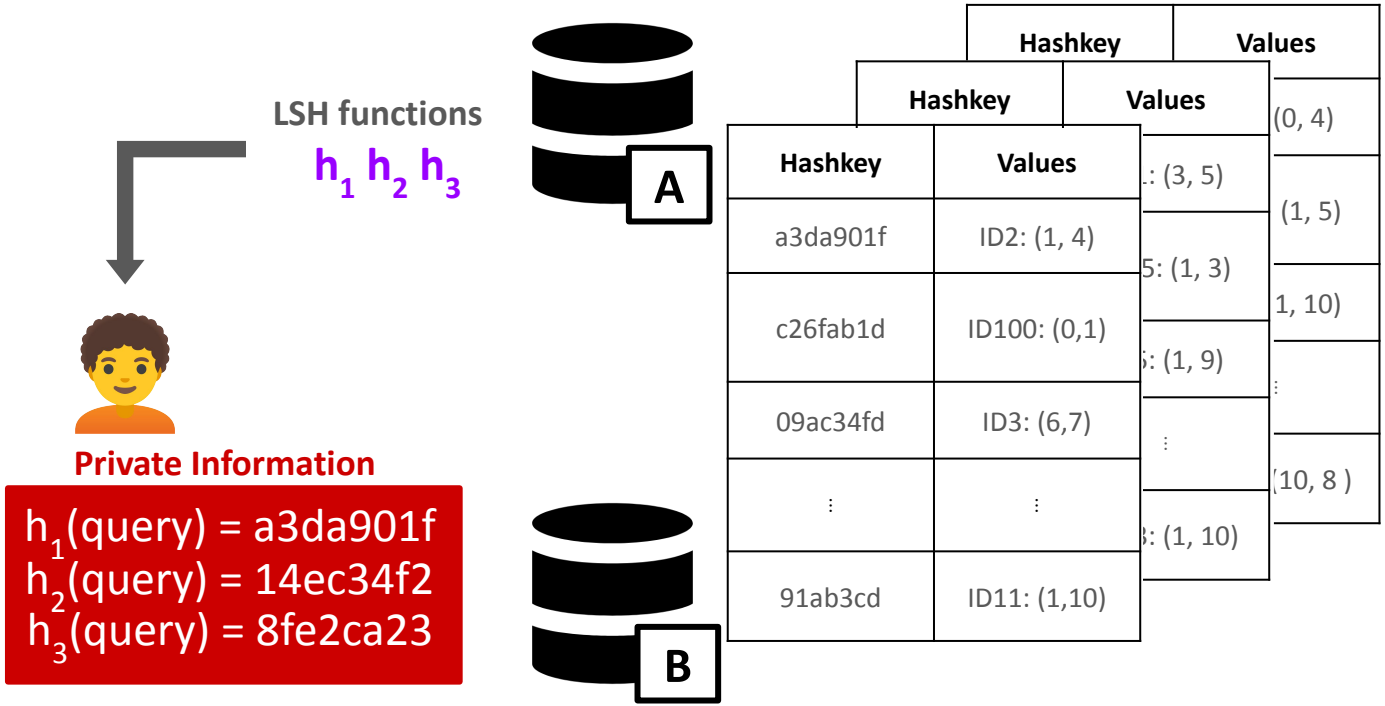

Challenge: querying the hash table privately

Q: How can the client retrieve the candidate from each table privately?

A: Using private information retrieval (PIR) [1].

- We use Distributed Point Functions [2] (DPFs) for efficiently querying hash tables in a two-server setting.

[1]: Gertner, Yael, et al. "Protecting data privacy in private information retrieval schemes." *Journal of Computer and System Sciences* 60.3 (2000): 592-629.
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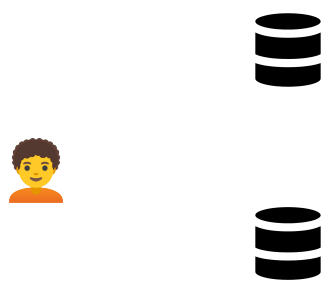

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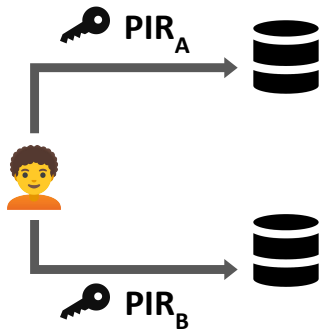

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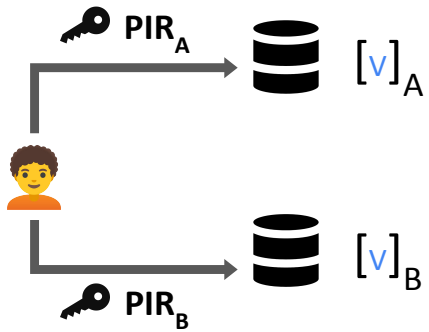

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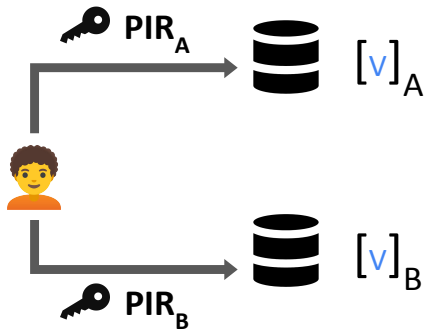

$$[v]_A + [v]_B = v \quad \text{(additive secret shares)}$$

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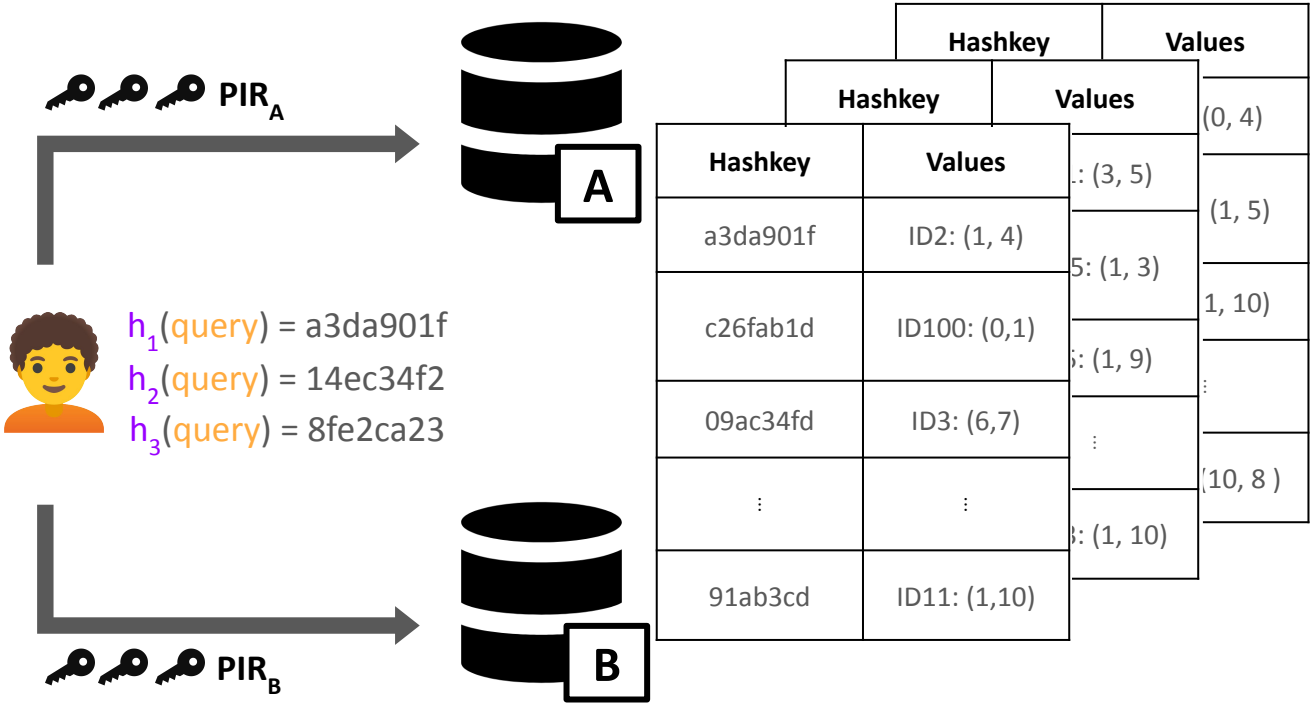
A strawman protocol

Step 1: Client uses LSH functions to find the hashkey

Step 2: Privately query hash tables using PIR



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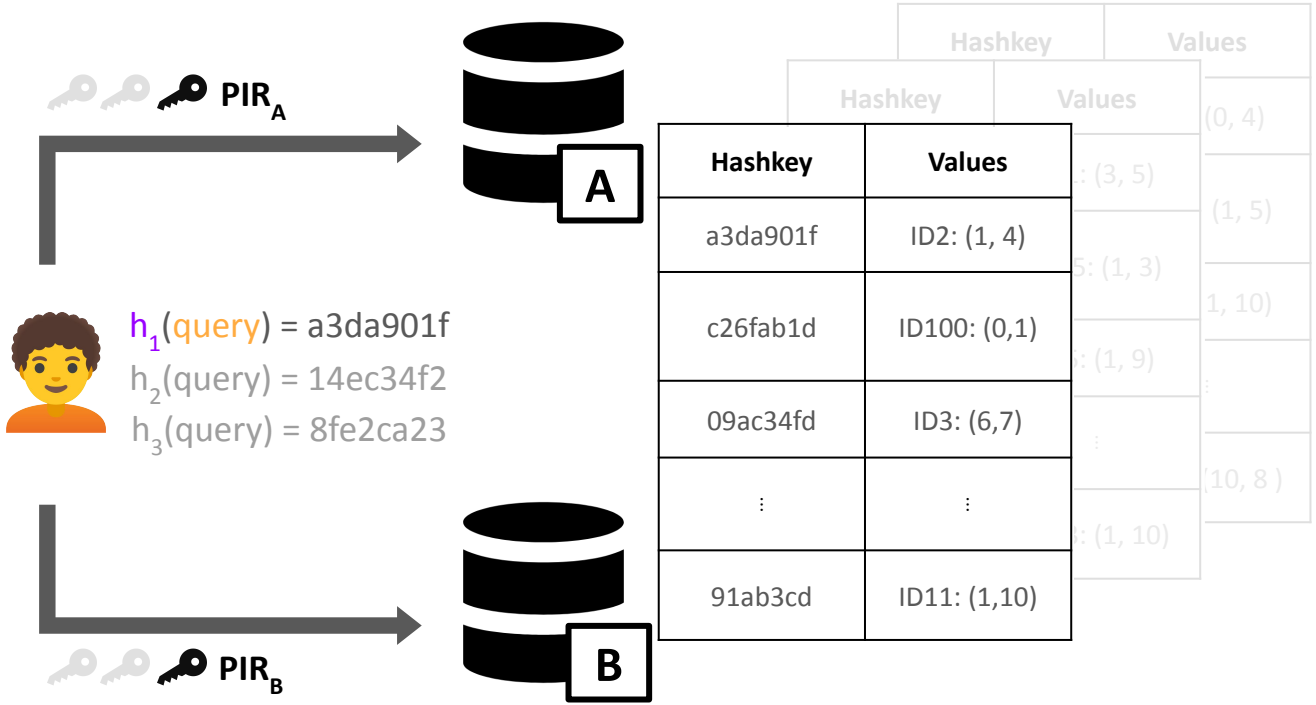

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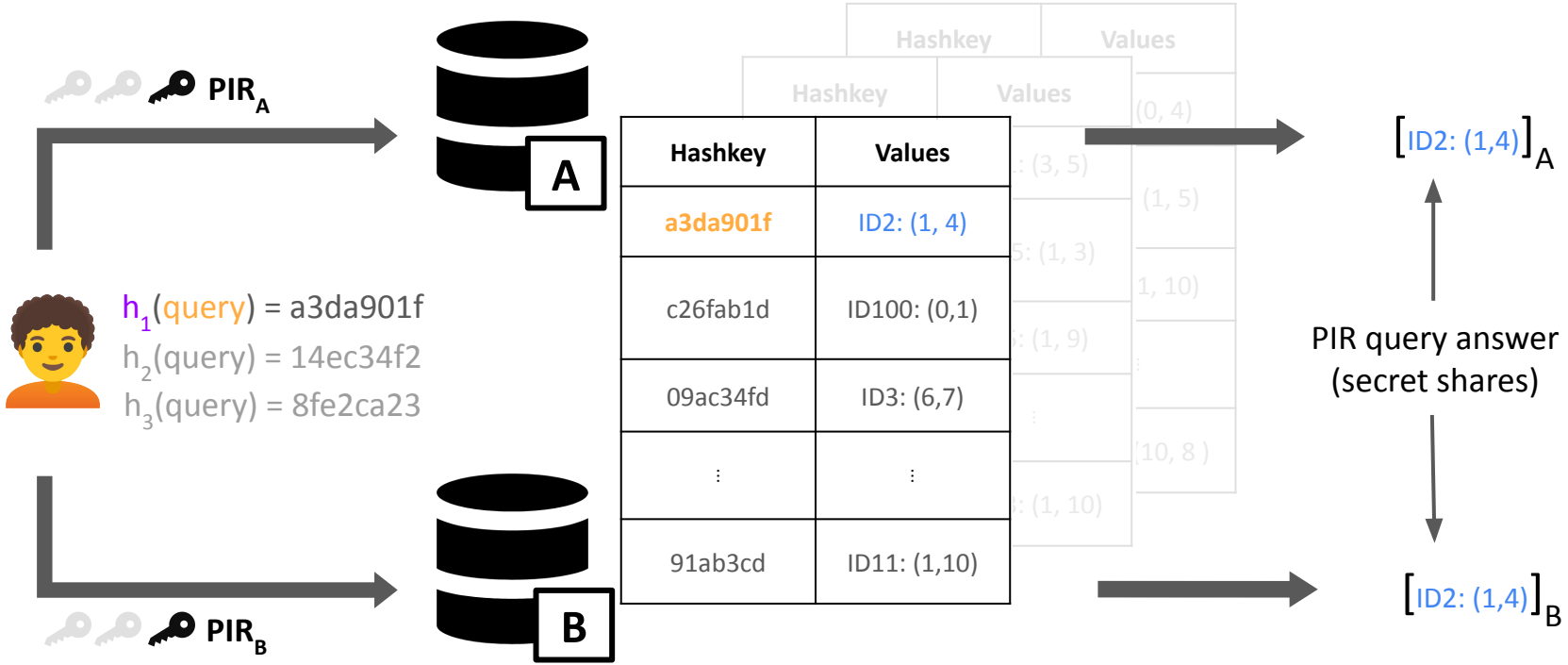

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Step 1: Client uses LSH functions to find the hashkey
Step 2: Privately query hash tables using PIR
Step 3: Client recovers the result

Query: (2,5)

Candidate set: { **ID2:** (1,4), **ID5694:** (2,4), **ID900:** (3,4), **ID101:** (6,5) }

(union over all tables)

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False positives

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***Near* Neighbors**

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Nearest **Neighbor**

Challenge: Database privacy

Candidate set: {**ID2:** (1,4), **ID5694:** (2,4), **ID900:** (3,4), **ID101:** (6,5)}

Challenge: Database privacy

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⚠️ **The candidate set leaks a lot about the database to the client!**

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The client learns:

- *All* near neighbors and their feature vectors.

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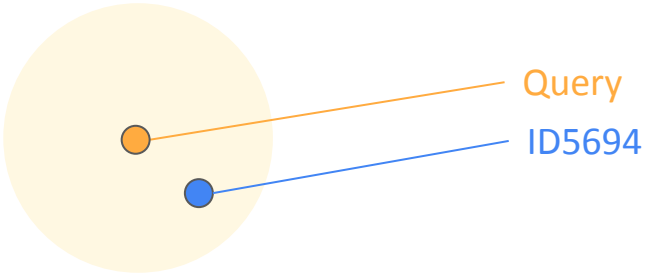
Baseline leakage



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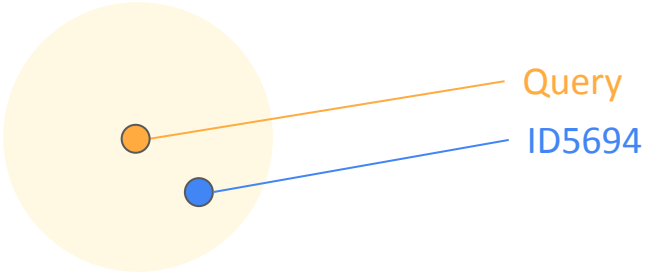
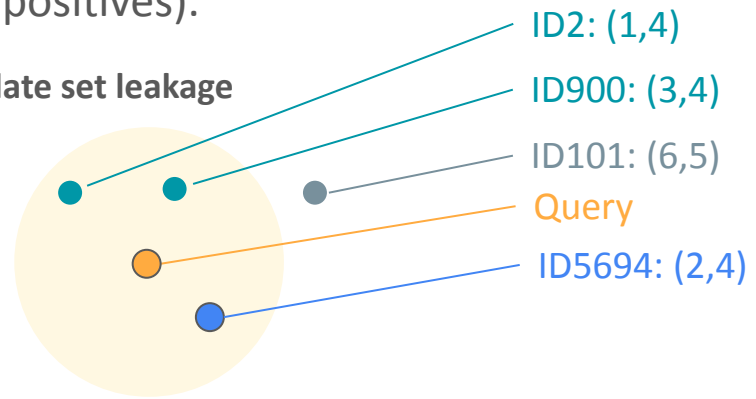


Candidate set leakage



Database Privacy with

Radix sorting

(hide all feature vectors by pruning without comparisons)

Pruning without comparisons

Main idea: use radix sorting for comparison-free sorting

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(i.e., let the hash function do the work of sorting by distance)

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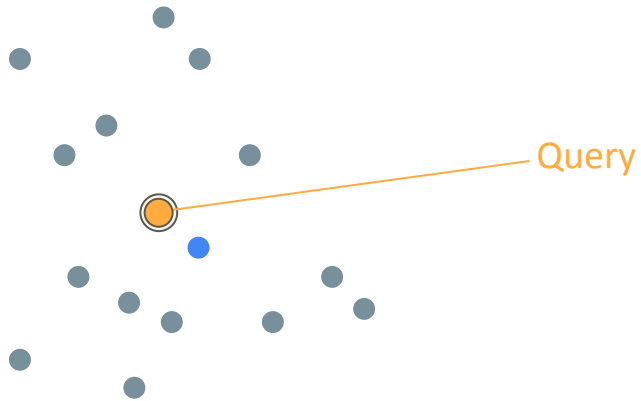
Table 1: radius = 0.1



Pruning without comparisons

Main idea: use radix sorting for comparison-free sorting

(i.e., let the hash function do the work of sorting by distance)

Table 1: radius = 0.1

Table 2: radius = 0.2

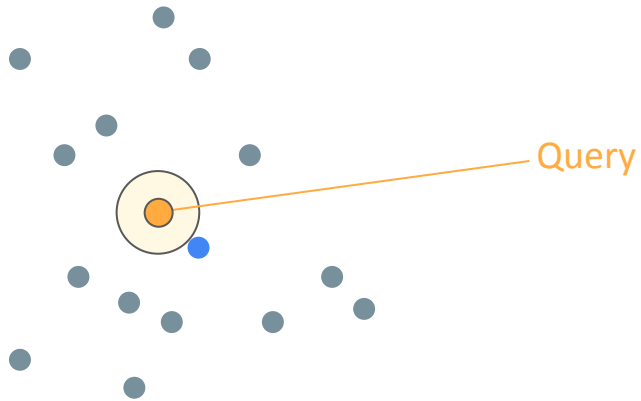


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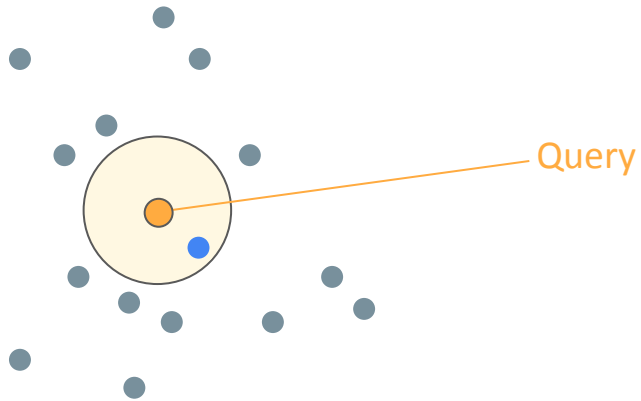
Table 1: radius = 0.1

Table 2: radius = 0.2

Table 3: radius = 0.3



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(i.e., let the hash function do the work of sorting by distance)

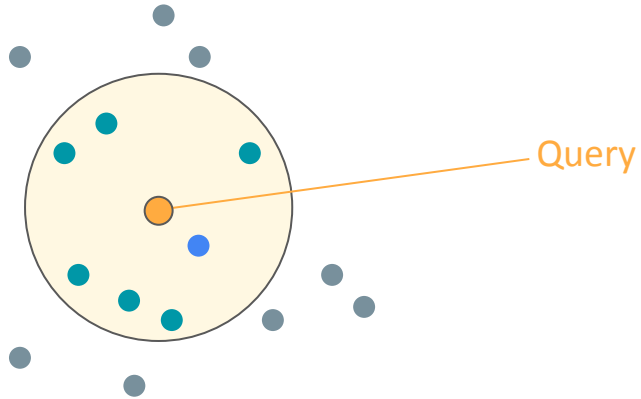
Table 1: radius = 0.1

Table 2: radius = 0.2

Table 3: radius = 0.3

…

Table 20: radius = 2.0



Pruning without comparisons

Candidates are now sorted by distance from the query!

Candidate Set = {0, 0, 0, ID5694, ID2, ID900, 0, ID101}

Pruning without comparisons

Candidates are now sorted by distance from the query!

Candidate Set = {0, 0, 0, ID5694, ID2, ID900, 0, ID101}

Nearest Neighbor

Pruning without comparisons

Candidates are now sorted by distance from the query!

Candidate Set = {0, 0, 0, ID5694, ID2, ID900, 0, ID101}

Nearest Neighbor

We no longer need to include the feature vectors (only the IDs)!

Pruning without comparisons

Candidates are now sorted by distance from the query!

Candidate Set = {0, 0, 0, ID5694, ID2, ID900, 0, ID101}

Nearest Neighbor

We no longer need to include the feature vectors (only the IDs)!

⚠️ **Still leaks many IDs from the database to the client.**

More database privacy with

Oblivious masking

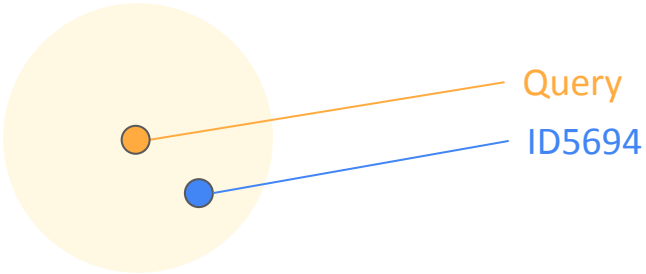
(hide all candidates except for the nearest neighbor)

Oblivious masking

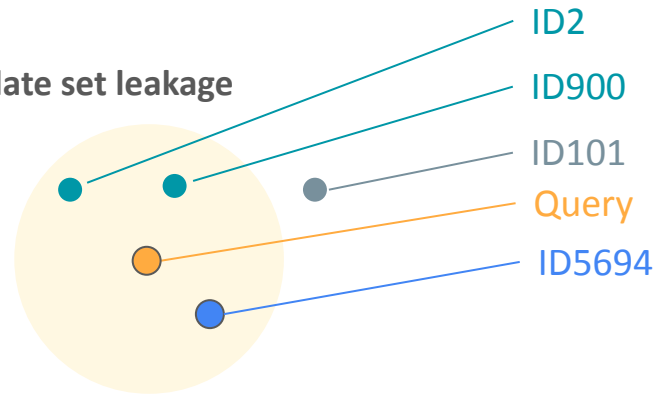
After processing the PIR queries, each server has a share of the candidate set:

$$[\text{Candidate Set}] = \{\ [0], [0], [0], [ID5694], [ID2], [0], [ID900], [ID101]\ \}$$

Baseline leakage



Candidate set leakage

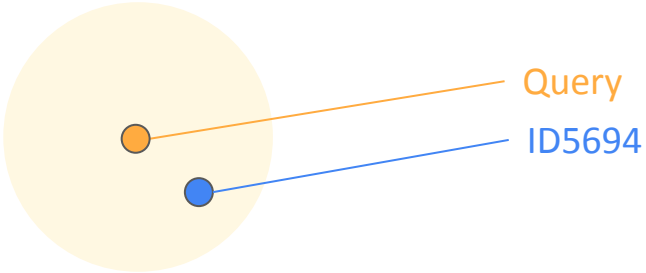


Oblivious masking

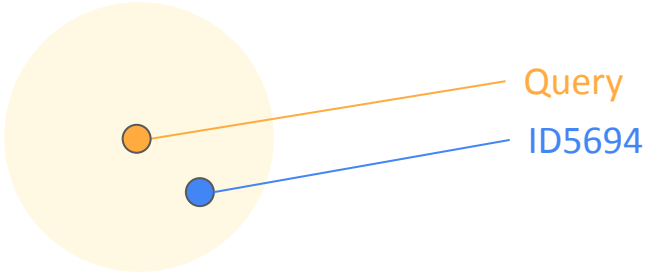
After processing the PIR queries, each server has a share of the candidate set:

[Candidate Set] = { [redacted] [ID5694], [redacted] }

Baseline leakage



Candidate set leakage



Oblivious masking

Observation: the nearest neighbor is always preceded by shares of 0s…

$$[\textbf{Candidate Set}] = \{ [0], [0], [0], [ID5694], [ID2], [0], [ID900], [ID101] \}$$

Oblivious masking

Observation: the nearest neighbor is always preceded by shares of 0s…

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

Algorithm 1: ObliviousMasking

Input: Secret-shared vector $[\boldsymbol{v}] \in \mathbb{F}_p^L$ and randomness rand.

Output: Secret-shared vector $[\boldsymbol{y}] = ([y_1], \ldots, [y_L]) \in \mathbb{F}_p^L$.

Procedure:

1: **for** $i \in \{1, \ldots, L\}$**:**

1.1: Sample $r_i \leftarrow$ rand.

1.2: Set $[\boldsymbol{y}_i] \leftarrow [\boldsymbol{v}_i] + r_i \cdot \left(\sum_{j=0}^{i-1} [\boldsymbol{v}_j] \right)$.

2: Output $\boldsymbol{y} \in \mathbb{F}_p^L$.

Oblivious masking

Step 1: Servers agree on common randomness ($r_1, r_2, \ldots, r_n$) e.g., with a short PRG seed.

$$[\textbf{Candidate Set}] = \{ [0], [0], [0], [ID5694], [ID2], [0], [ID900], [ID101] \}$$

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ \}$$

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ \quad \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add [SUM]$\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

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$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0] \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add [SUM]$\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0] + [0]r_2 \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add $[\text{SUM}]\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0], [0] + [0+0]r_3 \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add $[\text{SUM}]\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}] + [0+0+0]r_4 \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add $[\text{SUM}]\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

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$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}] + [0+0+0+\text{ID5694}]r_5 \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add $[\text{SUM}]\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}] + [(0+0+0+\text{ID5694})r_5] \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add $[\text{SUM}]\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

Step 2: Servers copy (and randomize) shares from **[Candidate Set]** to **[Candidate Set]**.

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\#\$@!\#*] \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add [SUM]$\cdot r_i$ to the new element in **[Candidate Set]**.

Oblivious masking

Step 1: Servers agree on common randomness (r_1, r_2, …, r_n) e.g., with a short PRG seed.

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$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\#\$@!\#*], [\$\%@!\&], \ldots \}$$

Rule: For each copied share, sum all preceding elements in **[Candidate Set]** and add [SUM]·r_i to the new element in **[Candidate Set]**.

Oblivious masking

All elements *after* the first non-zero element are **random**!

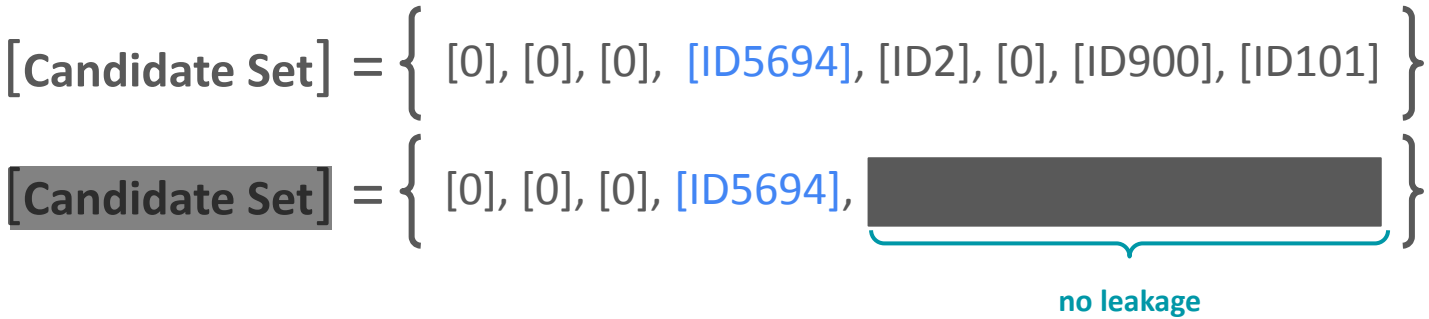
$$[\text{Candidate Set}] = \{\ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}]\ \}$$

$$[\text{Candidate Set}] = \{\ [0], [0], [0], [\text{ID5694}], [\#\$@!\#*], [\$\%@!\&], \ldots\ \}$$

Oblivious masking

All elements *after* the first non-zero element are **random**!

Result: The client learns nothing beyond the first non-zero element (i.e., the ANN)!



Oblivious masking

All elements *after* the first non-zero element are **random**!

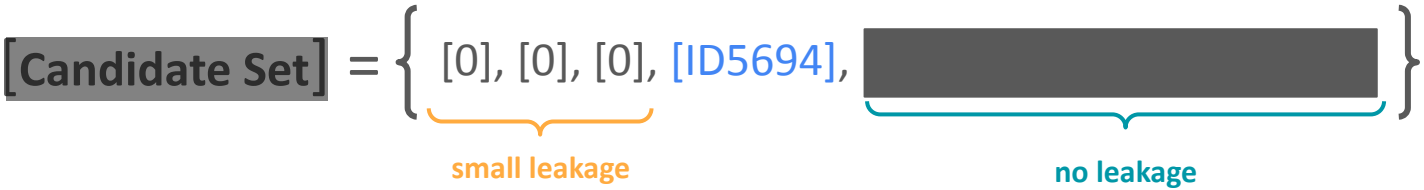
Result: The client learns almost nothing beyond the first non-zero element (i.e., the ANN)!

$$[\text{Candidate Set}] = \{ [0], [0], [0], [\text{ID5694}], [\text{ID2}], [0], [\text{ID900}], [\text{ID101}] \}$$

$$[\text{Candidate Set}] = \{ \underbrace{[0], [0], [0]}_{\text{small leakage}}, [\text{ID5694}], \underbrace{\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare}_{\text{no leakage}} \}$$

Oblivious masking

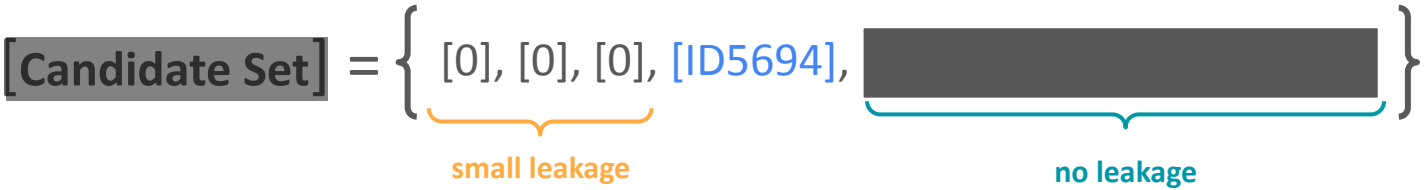
The radix bucket of the ANN leaks the approximate distance from the query.



Oblivious masking

The radix bucket of the ANN leaks the approximate distance from the query.

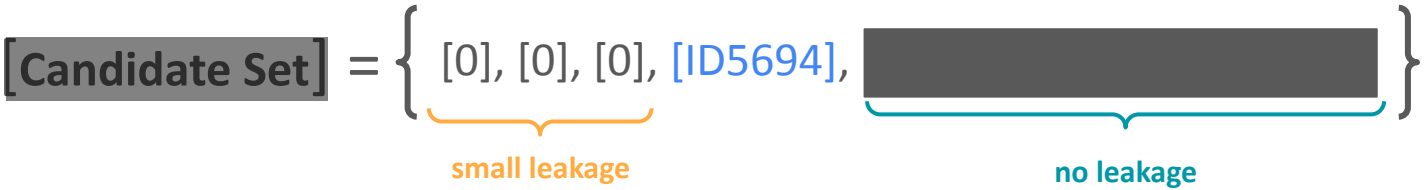
- **Asymptotically** optimal leakage!



Oblivious masking

The radix bucket of the ANN leaks the approximate distance from the query.

- **Asymptotically** optimal leakage!
- **In practice:** 2 to 15x more than baseline leakage.

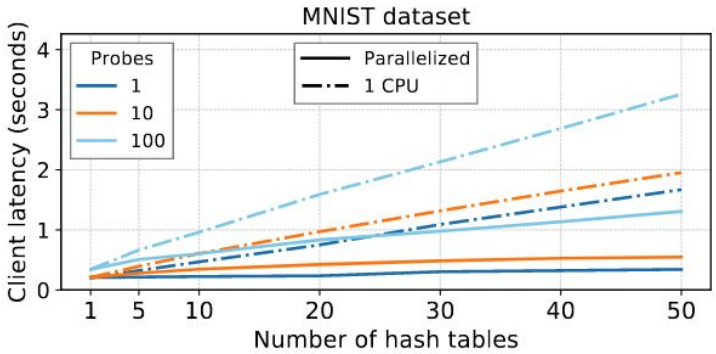


Evaluation

We evaluate on four real world datasets

- **Deep1B** (10 million items, 96 dimensions).
- **SIFT** (1 million items, 128 dimensions).
- **GIST** (1 million items, 916 dimensions).
- **MNIST** (60,000 items, 784 dimensions).

Efficiency on small datasets (MNIST; 60,000 items)

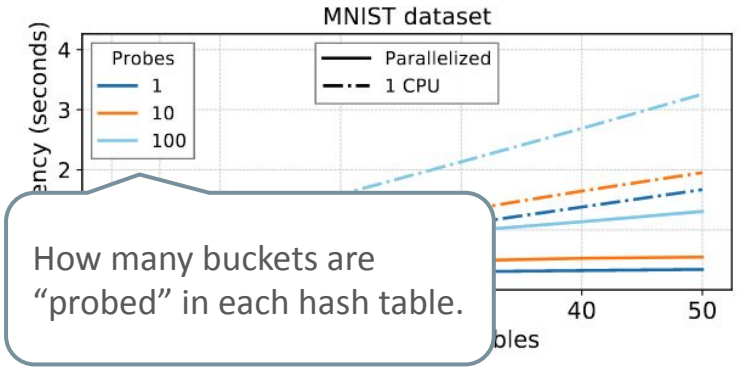


Server computation for > 95% accuracy (32 core servers)**:**

- **300 milliseconds per query.**

Communication: 800 KB between client and both servers.

Efficiency on small datasets (MNIST; 60,000 items)



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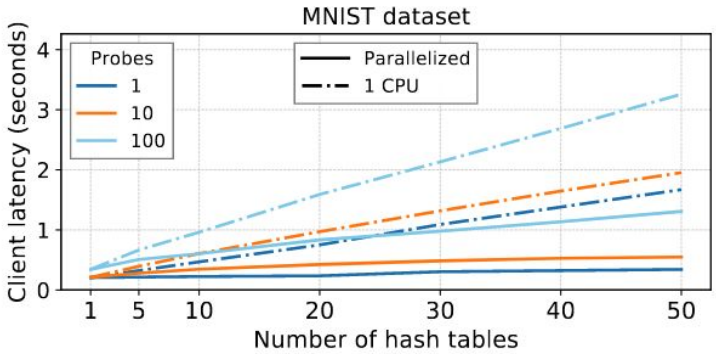

Server computation for > 95% accuracy (32 core servers)**:**

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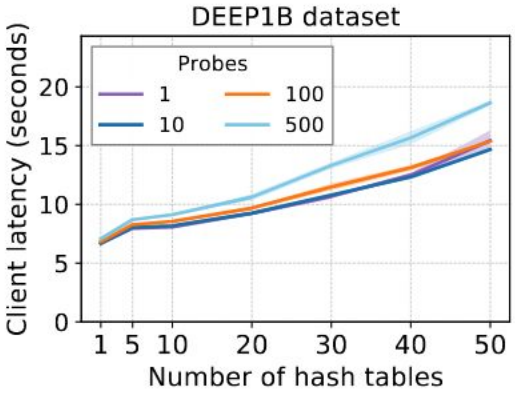
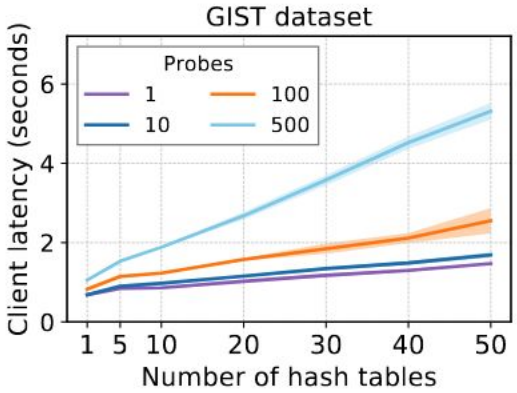
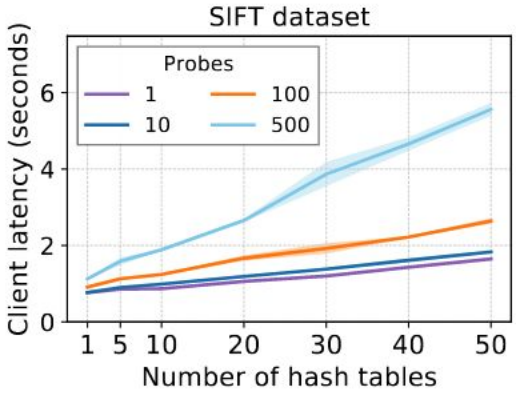
Efficiency on large datasets (1M to 10M items)





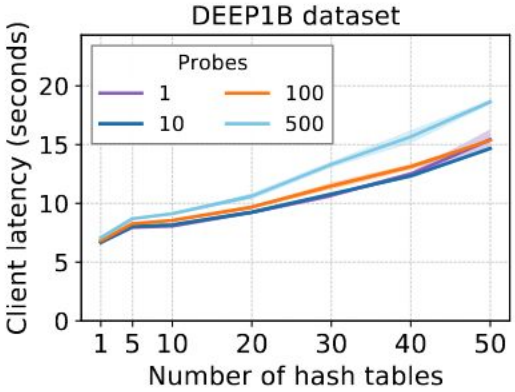


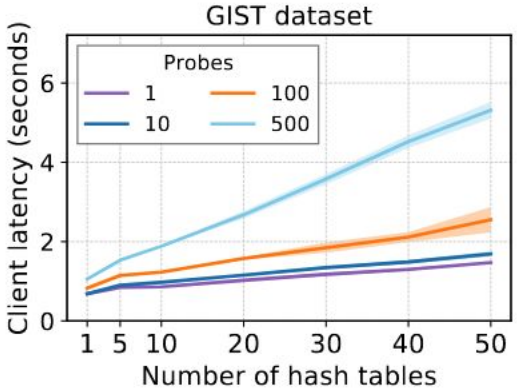
Server computation for > 95% accuracy (32 core servers)**:**

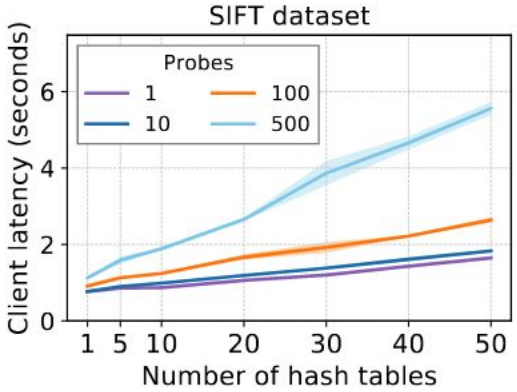
- **1.2 seconds per query** on 1M item datasets.
- **8 seconds per query** on 10M item datasets.

Communication: 1-2MB between client and both servers.

Efficiency on large datasets (1M to 10M items)



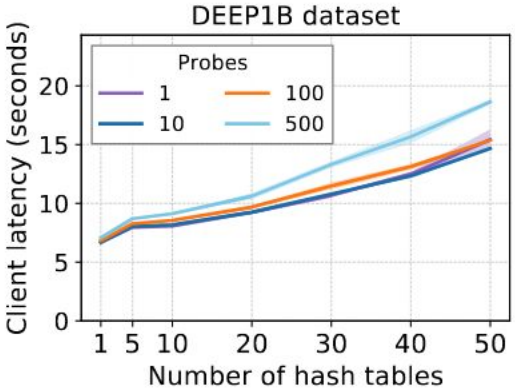


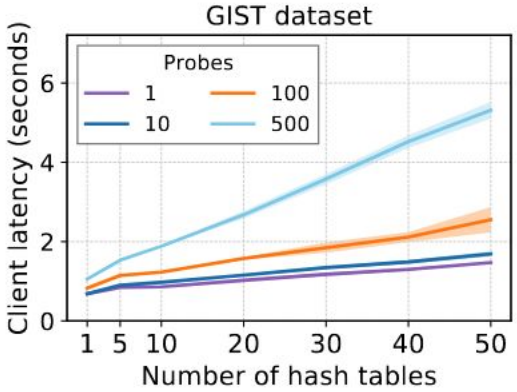


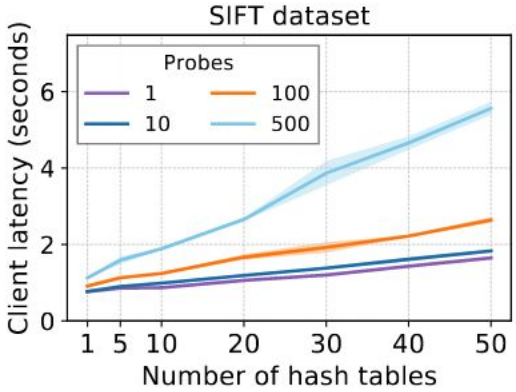
Server computation for > 95% accuracy (32 core servers)**:**

Five orders of magnitude less computation compared to FHE-based approaches.

Efficiency on large datasets (1M to 10M items)







1,000 to 3,000X less communication compared to two-party computation!

Communication: **1-2MB** between client and both servers.

Thank you!

Sacha Servan-Schreiber

with Simon Langowski and Srini Devadas

Full paper: ia.cr/2021/1157
Code: github.com/sachaservan/private-ann
Contact: 3s@mit.edu, slangows@mit.edu